
Phasors

Phase and Phasor Diagrams

What you'll learn in Module 5.

5.1 Phase Shift

Phase Shift in Common AC Components.

5.2 Phasors

Use of Phasors to Simplify Complex Waveform diagrams.

Relationship Between Waveform and Phasor Diagrams.

5.3 Phasor Diagrams

Using Phasor Diagrams to Show Phase Difference.

Drawing Phasor Diagrams.

5.4 Phasor Calculations

Using Phasor Diagrams to Calculate Unknown Values.

Scale Drawings

Pythagoras' Theorem

Trigonometry

5.5 Phasor Quiz

Phasor Quiz

Introduction

The AC circuits described in this module are some of the simplest in construction, consisting of only two or three components, but in their operation can seem almost magical, and are amazingly useful in so many ways. They might form the tuning circuit of a radio, and be used to improve the performance of amplifiers. They can greatly magnify alternating voltages or currents, and also reject radio interference. They may also be called "Tuned" or "Resonant" circuits and produce oscillations of a particular pitch or frequency in musical instruments. They are used to separate the high, low and mid range notes in a hi-fi speaker system. These same circuits perform functions like sensing a car as it approaches a traffic light, or help find buried precious metals with a metal detector.

As you study this and the following modules, notice that the circuits described use two main components, inductors and capacitors, which in many ways seem to have opposite and complimentary effects. It is the way they are connected in a circuit, as well as their individual electrical values that are key to the many uses of these circuits. Although in many cases these basic inductor/capacitor combinations may now be replaced by new and usually smaller ceramic components, **the basic principles of these LCR (inductor/capacitor/resistor) circuits are essential to understanding the operation of many electronic systems.**



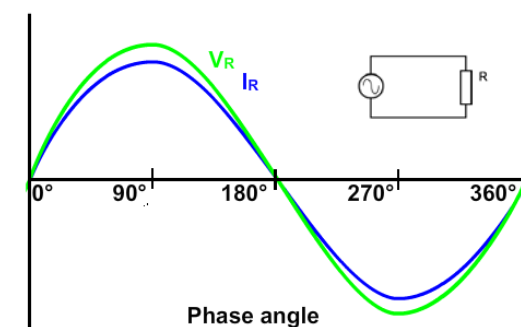
Module 5.1 Phase Shift

Fig 5.1.1 AC Phase Relationships in R, L and C

Resistance in AC Circuits

In purely resistive circuits, the current and voltage both change in the same way, and at the same time, as described in Module 4.1. This relationship is true, whether the applied voltage is direct or alternating. The main difference in AC circuits is that the voltage continues to change in a way that depends on the shape of the input wave. When a sine wave voltage is applied to a purely resistive circuit, it produces a sine wave (sinusoidal) current. Both waveforms attain their peak values at the same time, and pass through zero at the same time. Voltage and current in a purely resistive circuit are therefore said to be "IN PHASE" with each other.

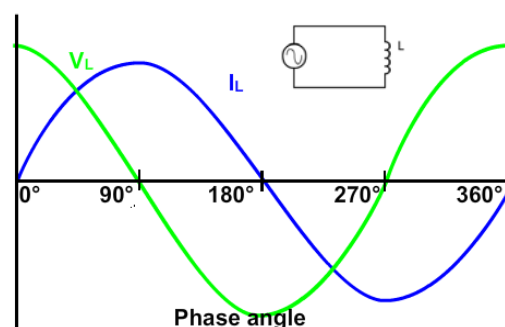
In a pure resistance, Voltage (V_R) & Current (I_R) are in phase



Inductance in AC Circuits

In a purely inductive circuit the voltage and current waveforms are not in phase. Inductance opposes change in current due to the back emf effect. This causes the current to reach its peak value some time after the voltage. So in an inductive circuit, current "LAGS" voltage.

In a pure inductance, Current (I_L) LAGS Voltage (V_L) by 90°



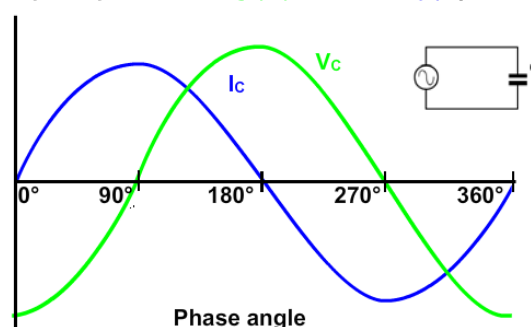
In DC circuits the current eventually settles to a steady state value, and the period of change prior to steady state depends on the time constant (i.e. the component values) of the circuit. In an AC circuit however, as the voltage is continually changing, the current also continues to change, and in a purely inductive circuit, the peak values of current occur a quarter of a cycle (90°) after those of the voltage.

In a circuit containing both inductance and resistance, which is usually the case as the inductor (a coil of wire) will have some internal resistance, the current will lag the voltage by an amount between practically 0° (nearly pure resistance) and almost -90° (nearly pure inductance). Since voltage and current no longer rise and fall together, a "PHASE SHIFT" is occurring in the circuit.

Capacitance in AC Circuits

Capacitance has the property of delaying changes in voltage as described in Module 4.3. That is, the applied voltage reaches steady state only after a time dictated by the time constant. In AC circuits voltage and current are changing continuously, and in a purely capacitive AC circuit the peak value of the voltage waveform occurs a quarter of a cycle after the peak value of the current. Therefore a phase shift is occurring in the capacitor, the amount of phase shift between voltage and current is $+90^\circ$ for a purely capacitive circuit, with the current LEADING the voltage. The opposite phase shift to an inductive circuit.

In a pure capacitance, Voltage (V_C) LAGS Current (I_C) by 90°



A very CIVIL relationship

One way to memorise these current/voltage (I/V) relationships in capacitors (C) and inductors (L) is to consider the positions of letters in the word CIVIL. The first three letters CIV indicate that in a capacitor (C), V lags (comes after) I, and the last three letters VIL indicate that I lags (comes after) V in an inductor (L).

Remember

CIVIL

I lags V in L

CIVIL

In C, V lags I

Module 5.2 Phasors

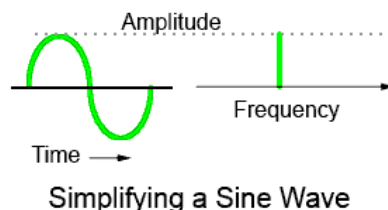
A better way to analyse complex waveform diagrams

In discussing phase shifts between currents and voltages in ac circuits, diagrams showing multiple waveforms such as those in module 5.1 could be used, but this involves a lot of complex drawing of waveforms which would, in many cases, be hard to interpret, and even more difficult to take measurements from. If calculations involving these waveforms were needed the waveforms would also need to be accurately drawn to scale.

A much more convenient method is to replace the sine wave drawings with phasor diagrams. Making use of a few facts about a sine wave, provides a much simpler way to represent a wave.

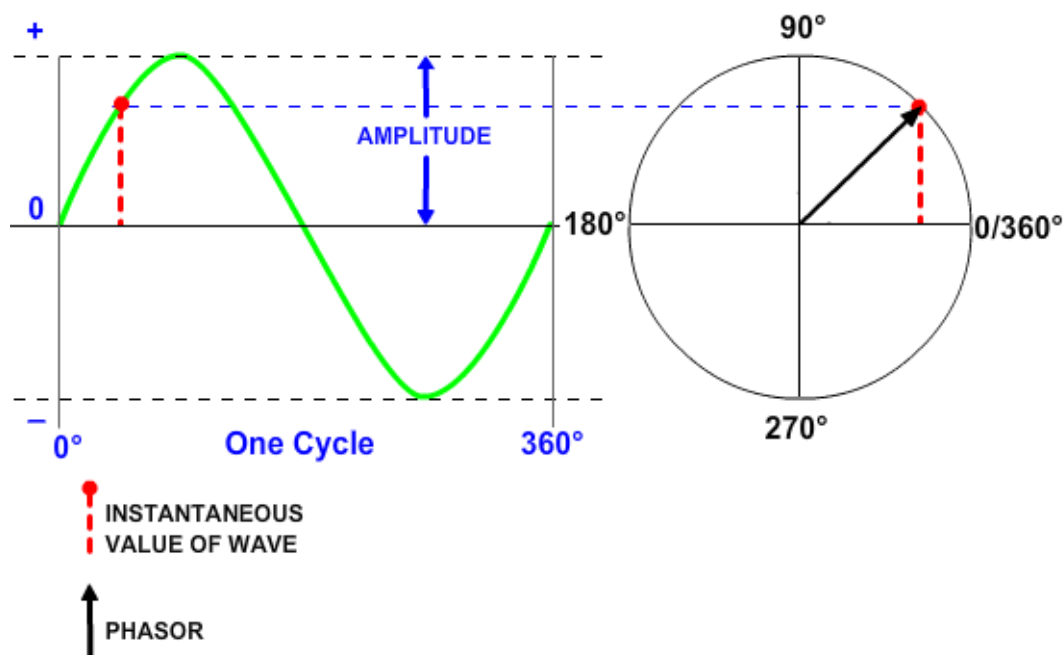
A sine wave has only one frequency, so this can be represented by simply choosing a suitable point on a horizontal line that represents a suitable range of frequencies.

The sine wave will have a particular amplitude, this can be represented by using a vertical line equal to the amplitude of the wave.



So the complex drawing of a sine wave shape can be simplified to just a couple of straight lines; one representing a frequency, and the other showing amplitude. The problem here however, is to draw a diagram that can represent a number of sine waves, each having the same frequency, but with different phase relationships to each other, which is where phasors are used.

Fig 5.2.1 How a Phasor Relates to a Sine Wave.



A phasor is a straight line drawn in such a way that its length is related to the amplitude of the sine wave represented, and its angular position relative to other phasors is related to the phase difference between the quantities.

The relationship between a phasor and a single sine wave is illustrated in fig 5.2.1. The phasor represented by the arrow is rotating in an anticlockwise direction about the centre origin point, describing the sine wave as it rotates.

Note that:

- a. At any point in time, the length of the red dotted line represents the instantaneous value of the wave.
- b. The length of the phasor represents the amplitude of the wave.
- c. The angle of the phasor gives the phase of the waveform. Increments in phasor angle in the circular diagram are equivalent to time or angle increments along the horizontal axis of the waveform diagram. So with this addition of angular information, the phasor gives a relatively simple way to show the complex relationships that exist between sine waves in an ac circuit.

Angular Velocity.

As a phasor is rotating, the speed at which it rotates about the origin point will depend on the frequency (f) of the wave represented. In one second the phasor will rotate through f revolutions, or through $f \times 360^\circ$ (since one revolution = 360°).

In calculations involving such rotation, it is more common to use the angular unit RADIAN (rad.) where $360^\circ = 2\pi$ rads. The phasor can therefore be said to rotate through $2\pi f$ radians per second. This is called the ANGULAR VELOCITY of the phasor, and is commonly represented by $2\pi f$ or alternatively by the symbol ω (omega).

Module 5.3 Phasor Diagrams

Phasor Diagrams Show Phase Difference

A phasor diagram is used to show the phase relationships between two or more sine waves having the same frequency. Section 5.2 showed a phasor continually rotating, but in use phasor diagrams are static. Imagine that the phasors are rotating in an anticlockwise (counter clockwise) direction.

Every phasor in the diagram will have the same angular velocity because they represent sine waves of identical frequency. The length of the each phasor arm is directly related to the amplitude of the wave it represents, and the angle between the phasors is the same as the angle of phase difference between the sine waves.

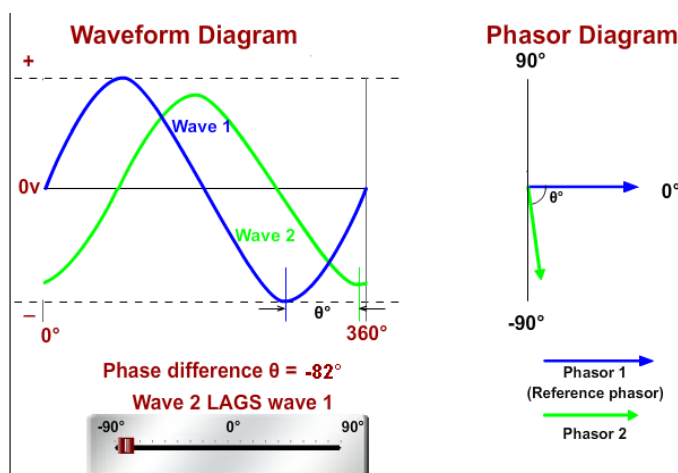


Fig 5.3.1 Showing Phase Relationship with Phasors

Fig 5.3.1 shows how a phasor diagram is used to illustrate the phase difference between waves 1 and 2.

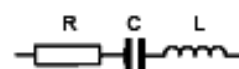
The main value of phasor diagrams is that they can be used, not only to represent waveform diagrams, but also in carrying out calculations involving ac waves. The calculations can involve any of the common values (RMS, peak values, phase angles etc.) and will be much quicker and easier than performing the calculations on waveform diagrams.

It can be seen from Fig 5.3.1 that the waveform diagrams can be represented in phasor form without any loss of information. Phasors can be used to calculate unknown values in different types of circuits where capacitors, resistors and inductors may be connected in series or in parallel to achieve different effects, provided some basic rules are followed.

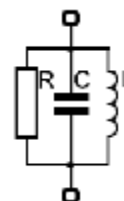
Five Rules for Drawing Phasor Diagrams.

Rule 1. The length of the phasor is directly proportional to the amplitude of the wave depicted.

Rule 2. In circuits which have combinations of L, C & R in SERIES (studied in Module 8)



it is customary to draw the phasor representing CURRENT horizontally, and call this the REFERENCE phasor. This is because the current in a series circuit is common to all the components.



Rule 3. In parallel circuits, (studied in Module 9) the phasor representing the SUPPLY VOLTAGE is always drawn in the REFERENCE direction. This is because in a parallel circuit it is the supply voltage that is common to all components.

Rule 4. The direction of rotation of all phasors is considered to be ANTICLOCKWISE.

Rule 5. In any one diagram, the same type of value (RMS, peak etc.) is used for all phasors, not a mixture of values.

Module 5.4 Phasor Calculations

Find Unknown Values Using Phasor Diagrams

There are several ways of using phasor diagrams to find out important data about LCR circuits. For example: "What supply voltage will be needed to ensure a particular voltage across the inductor in a circuit?" or "What would be the phase difference between the supply voltage and supply current at a particular frequency of operation?" and many others. The three methods described below introduce different ways of using phasor diagrams to find unknown quantities.

Study these three methods and once you are familiar with these methods, you can choose the best one to solve any particular problem involving the values and phase angles of AC currents and voltages.

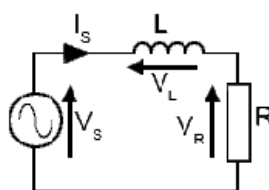
To help you on the right track why not download our "Maths Tips" booklet from the Downloads page our website <http://www.learnabout-electronics.org>, which shows you how to use your calculator to calculate angles and values using trigonometry functions and get the right answer every time.

No scientific calculator? The "Maths Tips" booklet explains what you need and don't need from a calculator. If you don't want to buy a scientific calculator, you can always pick up a free one from the net. PC users can try Calc98 from <http://www.calculator.org/download.html>. Whichever calculator you choose remember that you should read its instructions to become familiar with the working methods you should use as these do vary from calculator to calculator.

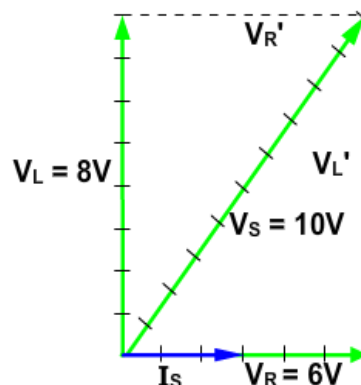
Method 1 Scale Drawing

Using a phasor diagram drawn to scale, find the supply voltage V_s when $V_R = 6V$ and $V_L = 8V$.

Because the components of the circuit are connected in series, I_s is common to all components, so I_s will be the reference phasor. It is drawn at 0° (horizontally to the right of the origin point). As the problem only requires voltages, the scale of I_s does not matter.



Suppose in the above circuit, the voltage across the resistor (V_R) is 6V and the voltage across the inductor (V_L) is 8V,
What is the value of the supply voltage (V_s)?



A phasor is drawn 6 units (cm, inches etc.) long. Because both resistor current and

voltage are always in phase with each other, V_R is in phase with the reference phasor I_s . The value is shown as $V_R = 6V$. (the small tick marks on the phasor are only given here to show scale.)

In a pure inductor, voltage (V_L) leads current (I_s) by 90° , so an 8V phasor is drawn vertically from the common point of origin.

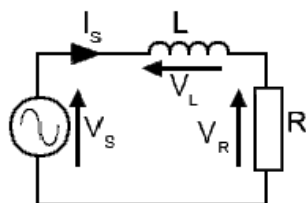
A line, the same length as, and parallel to V_R is drawn from the top of V_L to form the top of a rectangle (V_R').

The right side of the rectangle (V_L') is drawn from the tip of V_R , the same length as, and parallel to (V_L)

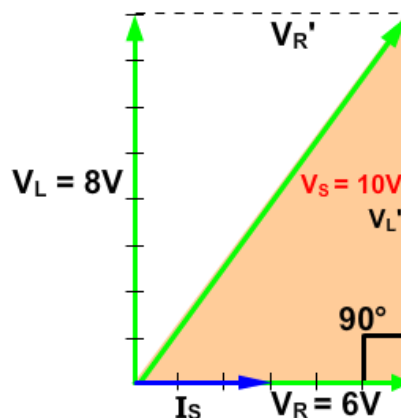
A phasor showing V_s can now be drawn from the origin point, diagonally to the opposite corner of the rectangle.

The measured length of this phasor will be the phasor sum of V_R and V_L , which in this case will be 10V.

Method 2 Pythagoras' Theorem



Suppose in the above circuit, the voltage across the resistor (V_R) is 6V and the voltage across the inductor (V_L) is 8V,
What is the value of the supply voltage (V_S)?

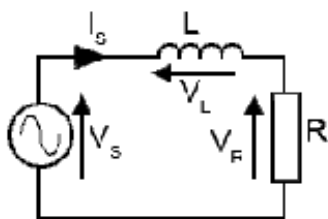


Because, in this example, the area within V_S , V_L and V_R is a right angle triangle, the length of the Hypotenuse (V_S) can be found using Pythagoras' Theorem, which states:

The square of the hypotenuse (V_S in this example) of a right angle triangle is equal to the sum of the squares of the two adjacent sides (V_L and V_R).

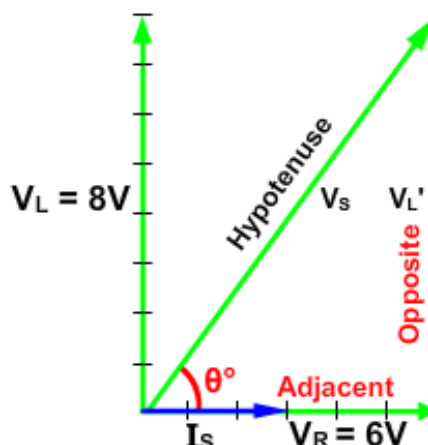
1. The formula: $V_S^2 = V_L^2 + V_R^2$ (using the symbols from the phasor diagram).
2. $V_S^2 = 8^2 + 6^2$ (replace the symbols with the known values).
3. $V_S^2 = 64 + 36 = 100$ (work out the value of V_S^2).
4. $V_S = \sqrt{100}$ (find the square root of V_S^2 to give V_S)
5. The Answer: **$V_S = 10V$** (remember the units you are working in - Volts in this example).

Method 3 Using the Inverse Trigonometry Functions



Suppose in the above circuit, the voltage across the resistor (V_R) is 6V and the voltage across the inductor (V_L) is 8V,

What is the value of the phase difference (angle θ) between (V_s) and (I_s)



The formula for finding an angle depends on which sides of the triangle have a known value. The choice for finding the unknown angle θ (in degrees) is between:

$$\theta^\circ = \sin^{-1} (\text{Opposite} / \text{Hypotenuse})$$

$$\theta^\circ = \cos^{-1} (\text{Adjacent} / \text{Hypotenuse})$$

$$\theta^\circ = \tan^{-1} (\text{Opposite} / \text{Adjacent})$$

Even if the value of the hypotenuse (V_s) is not yet known (from Method 1 or 2) the \tan^{-1} formula can be used to find the angle θ .

$$\theta^\circ = \tan^{-1} (\text{Opposite} / \text{Adjacent})$$

$$\theta^\circ = \tan^{-1} (8 / 6)$$

$$\theta^\circ = \tan^{-1} 1.33$$

$$\theta^\circ = 53.13^\circ \text{ (} 53^\circ \text{ would normally be near enough)}$$

To help you on the right track why not download our "Maths Tips" booklet, available from http://www.learnabout-electronics.org/ac_theory/ac_ccts_54.php which shows you how to use your calculator to calculate angles and values using trigonometry functions and get the right answer every time.

No scientific calculator? The "Maths Tips" booklet explains what you need and don't need from a calculator. If you don't want to buy a scientific calculator, you can always pick up a free one from the net. PC users can try Calc98 from www.calculator.org/download.html. Whichever calculator you choose remember that you should read its instructions to become familiar with the working methods you should use, as these do vary from calculator to calculator.

Module 5.5 Phasor Quiz

What you should know.

After studying Module 5, you should be able to:

Describe phase relationships in sine waves applied to capacitive, inductive and resistive circuits.

Describe the representation of sine waves using phasors.

Interpret phasor diagrams relating to voltage current and phase relationships in ac circuits.

Calculate values in ac circuits using phasor diagrams.

Try our quiz, based on the information you can find in Module 5. Check your answers on line at:
http://www.learnabout-electronics.org/ac_theory/ac_ccts_55.php

1.

In an inductor, an increase in the value of the internal resistance has the effect of:

- a) Increasing the inductance.
- b) Decreasing the phase difference between the inductor voltage (V_L) and current (I_L).
- c) Decreasing the inductance.
- d) Increasing the phase difference between the inductor voltage (V_L) and current (I_L).

2.

Complete the sentence: In a capacitor...

- a) Current leads voltage by 90° .
- b) Voltage leads current by 90° .
- c) Current Lags Voltage by between 0° and 90° .
- d) Voltage leads current by between 0° and 90° .

3.

Which property of a sine wave does the length of a phasor represent?

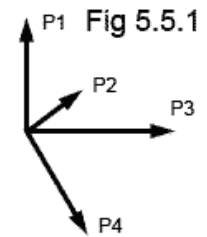
- a) Frequency.
- b) Phase.
- c) Amplitude.
- d) Instantaneous value.

Continued:

4.

Refer to Fig. 5.5.1: Which of the following statements about Fig 5.5.1 is true?

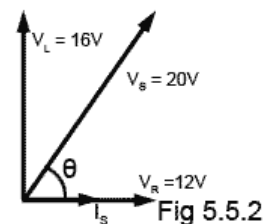
- a) All the waves depicted have the same frequency and same amplitude.
- b) All the waves depicted have the same phase and same frequency.
- c) All the waves depicted have the same shape and same frequency.
- d) All the waves depicted have the same phase and same shape.



5.

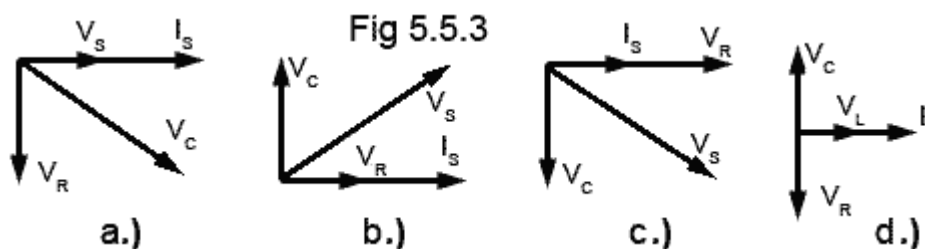
Refer to Fig 5.5.2: What is the phase difference θ between the supply voltage and the supply current?

- a) 53°
- b) 37°
- c) 30°
- d) 59°



6.

Refer to Fig 5.5.3: Which of the phasor diagrams describes a CR series Circuit?



7.

Refer to Fig 5.5.1: If phasor P2 represents 5V and phasor P3 represents 11V, what is the approximate angle between P2 and P3?

- a) 27°
- b) 67°
- c) 71°
- d) 63°

8.

In a parallel LCR circuit, which of the following properties is used as the reference phasor?

- a) Supply current (I_s).
- b) Reactive voltage ($V_C - V_L$).
- c) Supply voltage (V_s).
- d) Reactive voltage ($V_L - V_C$).

Continued:

9.

A phasor representing a 1kHz sine wave will rotate through how many complete radians in one second?

- a) 360,000 b) 6283 c) 1000 d) 3142

10.

In a phasor diagram, the frequency of each of the waves shown by the phasors is:

- a) The same for all waves, and shown by the point of origin of the phasors.
- b) Different for each wave, and proportional to the different lengths of the phasors.
- c) Different for each wave, and shown by the change in angle of the phasors.
- d) Not shown.