1

THEORY*

This discussion of thermodynamics is limited to the processes that are involved in the compression of gases in a positive displacement compressor of the reciprocating type. A *positive displacement compressor* is a machine that increases the pressure of a definite initial volume of gas, accomplishing the pressure increase by volume reduction. Only with a knowledge of basic laws and their application can one understand what is happening in a compressor and thus properly solve any compression problem.

The definitions and units of measurement given at the end of this chapter should either be known to the reader or be reviewed thoroughly before beginning.

1.1 SYMBOLS

The following symbols (based on pounds, feet, seconds, and degrees Fahrenheit) are used in this discussion of positive displacement compressor theory:

c	cylinder clearance, % or decimal
c_p	specific heat-constant pressure, Btu/°F-lb
c_v	specific heat-constant volume, Btu/°F-lb
CE	compression efficiency, %
k	ratio of specific heats, dimensionless
M	molecular weight (MW), dimensionless
ME	mechanical efficiency, %
N	number of moles, dimensionless

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* Developed and contributed by Dresser-Rand Company, Olean, N.Y. Based on Ingersoll-Rand Form 3519-D.

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$N_{a,b,c}$	moles of constituents, dimensionless
p	pressure, psia
$p_{a,b,c}$	partial pressure of constituents, psia
p_a	partial air pressure, psia
p_c	critical pressure (gas property), psia
p_r	reduced pressure, dimensionless
p_s	saturated vapor pressure, psia or in. Hg
p_v	partial vapor pressure, psia or in. Hg
psia	lb/in^2 absolute, psi
psig	lb/in^2 gauge, psi
P_t	theoretical horsepower, (work rate), hp
Q	heat, Btu
r	ratio of compression per stage, dimensionless
r_t	ratio of compression—total, dimensionless
R_0	universal or molar gas constant, ft-lb/mol-°R (1545 when p is in lb/ft^2)
R'	specific gas constant, ft-lb/lb-°R
RH	relative humidity, %
s	number of stages of compression, dimensionless
S	entropy, Btu/lb-°F
SH	specific humidity, lb moisture/lb dry gas
SPT	standard pressure and temperature, 14.696 psia and 60°F
T	absolute temperature, °R
T_c	critical temperature, °R
T_r	reduced temperature, dimensionless
v	specific volume, ft^3/lb
$v_{a,b,c}$	partial volume of constituents, ft^3/lb
v_r	pseudo-specific reduced volume, ft^3/lb
V	total volume, ft^3
VE	volumetric efficiency, %
W	weight, lb
W_a	weight of dry air in a mixture, lb
W_v	weight of vapor in a mixture, lb
$W_{a,b,c}$	weight of constituents in a mixture, lb
Z	compressibility factor, dimensionless
η_v	volumetric efficiency, %

1.2 HOW A COMPRESSOR WORKS

Every compressor is made up of one or more *basic elements.* A single element, or a group of elements in parallel, comprises a *single-stage* compressor. Many compression problems involve conditions beyond the practical capability of a single compression stage. Too great a *compression ratio* (absolute discharge pressure divided by absolute intake pressure) causes excessive discharge temperature and other design problems. It therefore may become necessary to combine elements or groups of elements in series to form a *multistage unit*, in which there will be two or more steps of compression. The gas is frequently cooled between stages to reduce the temperature and volume entering the following stage.

Note that each stage is an individual basic compressor within itself. It is sized to operate in series with one or more additional basic compressors, and even though they may all operate from one power source, each is still a separate compressor.

The basic reciprocating compression element is a single cylinder compressing on only one side of the piston (*single-acting*). A unit compressing on both sides of the piston (*double-acting*) consists of two basic single-acting elements operating in parallel in one casting.

The reciprocating compressor uses automatic spring-loaded valves that open only when the proper differential pressure exists across the valve. Inlet valves open when the pressure in the cylinder is slightly below the intake pressure. Discharge valves open when the pressure in the cylinder is slightly above the discharge pressure.

Figure 1.1 shows the basic element with the cylinder full of a gas, say, atmospheric air. On the theoretical *p–V* diagram (indicator card), point 1 is the start of compression. Both valves are closed.

Figure 1.2 shows the compression stroke, the piston having moved to the left, reducing the original volume of air with an accompanying rise in pressure. Valves remain closed. The *p–V* diagram shows compression from point 1 to point 2, and the pressure inside the cylinder has reached that in the receiver.

Figure 1.3 shows the piston completing the delivery stroke. The discharge valves opened just beyond point 2. Compressed air is flowing out through the discharge valves to the receiver. After the piston reaches point 3, the discharge valves will close, leaving the clearance space filled with air at discharge pressure.

During the expansion stroke (Fig. 1.4) both the inlet and discharge valves remain closed, and air trapped in the clearance space increases in volume, causing a reduction in pressure. This continues as the piston moves to the right, until the cylinder pressure drops below the inlet pressure at point 4.

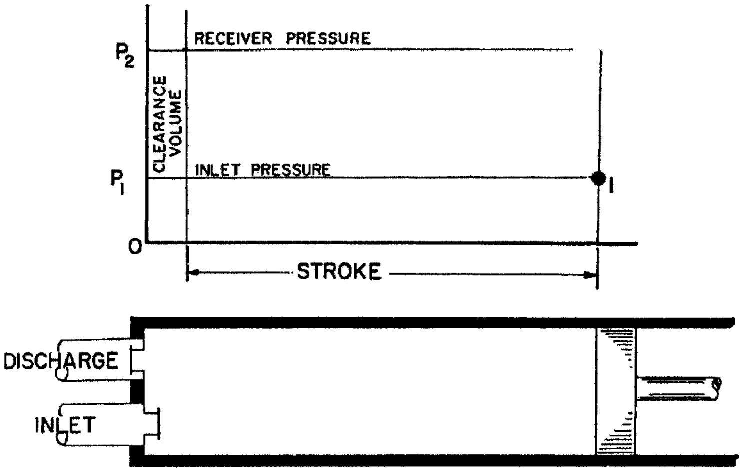

FIGURE 1.1 Basic compressor element with the cylinder full of gas. On the theoretical *p–V* diagram (indicator card), point 1 is the start of compression. Both valves are closed. *(Dresser-Rand Company, Painted Post, N.Y.)*

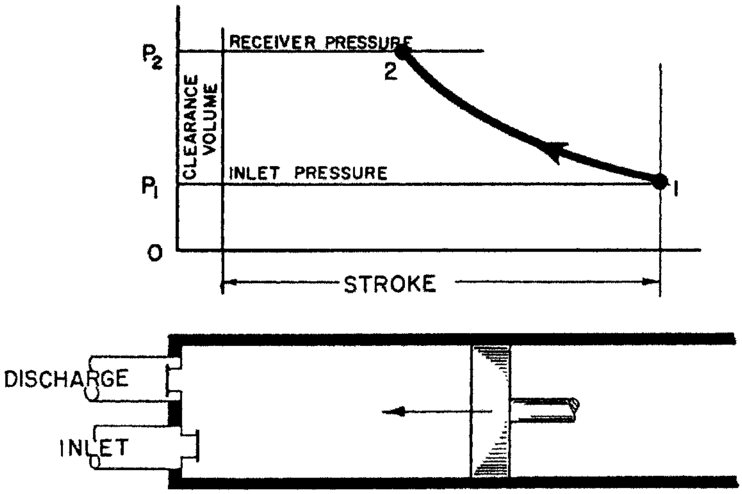

FIGURE 1.2 Compression stroke. The piston has moved to the left, reducing the original volume of gas with an accompanying rise in pressure. Valves remain closed. The *p–V* diagram shows compression from point 1 to point 2 and the pressure inside the cylinder has reached that in the receiver. *(Dresser-Rand Company, Painted Post, N.Y.)*

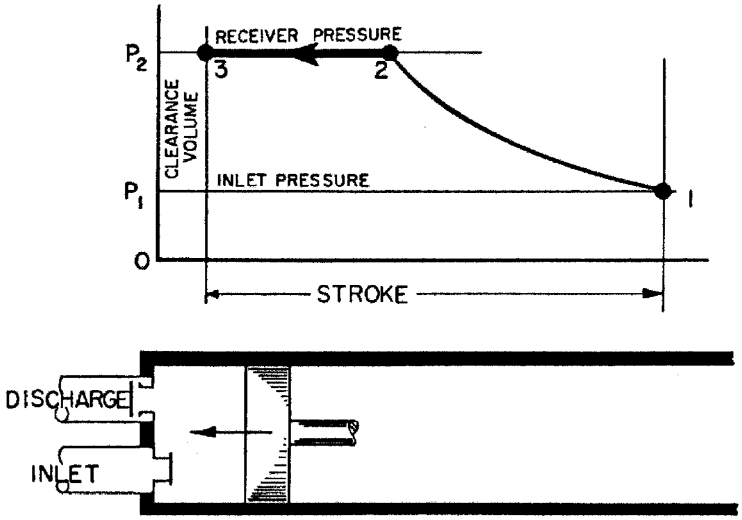

FIGURE 1.3 The piston is shown completing the delivery stroke. The discharge valves opened just beyond point 2. Compressed air is flowing out through the discharge valves to the receiver. *(Dresser-Rand Company, Painted Post, N.Y.)*

The inlet valves will now open, and air will flow into the cylinder until the end of the reverse stroke at point 1. This is the intake or suction stroke, illustrated by Fig. 1.5. At point 1 on the *p–V* diagram, the inlet valves will close and the cycle will repeat on the next revolution of the crank.

In an elemental two-stage reciprocating compressor the cylinders are proportioned according to the total compression ratio, the second stage being smaller because the gas,

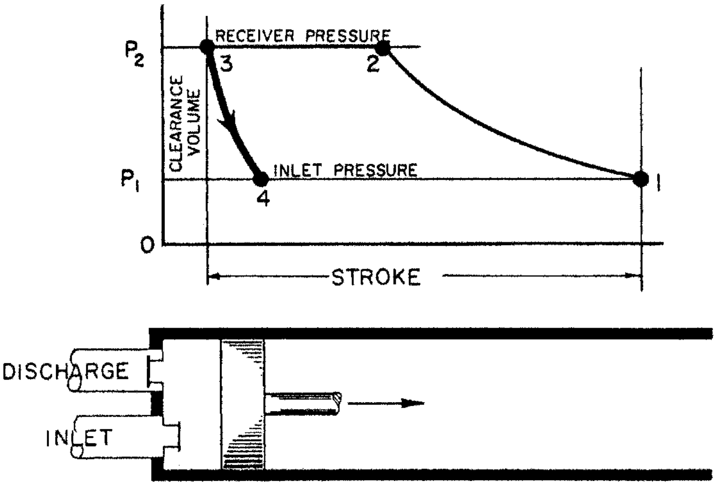

FIGURE 1.4 During the expansion stroke shown, both the inlet and discharge valves remain closed and gas trapped in the clearance space increases in volume, causing a reduction in pressure. *(Dresser-Rand Company, Painted Post, N.Y.)*

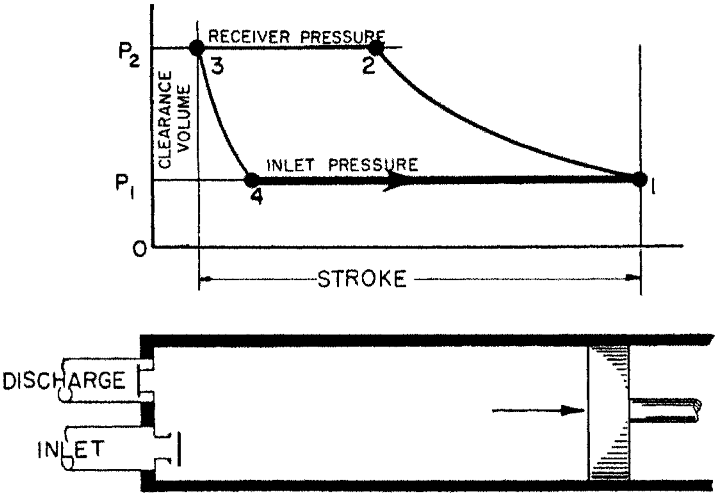

FIGURE 1.5 At point 4, the inlet valves will open and gas will flow into the cylinder until the end of the reverse stroke at point 1. *(Dresser-Rand Company, Painted Post, N.Y.)*

having already been partially compressed and cooled, occupies less volume than at the first-stage inlet. Looking at the p–V diagram (Fig. 1.6), the conditions before starting compression are points 1 and 5 for the first and second stages, respectively; after compression, conditions are points 2 and 6, and after delivery, points 3 and 7. Expansion of gas trapped in the clearance space as the piston reverses brings points 4 and 8, and on the intake stroke the cylinders are again filled at points 1 and 5 and the cycle is set for repetition. Multiple staging of any positive displacement compressor follows this pattern.

Certain laws that govern the changes of state of gases must be thoroughly understood. Symbols were listed in Section 1.1.

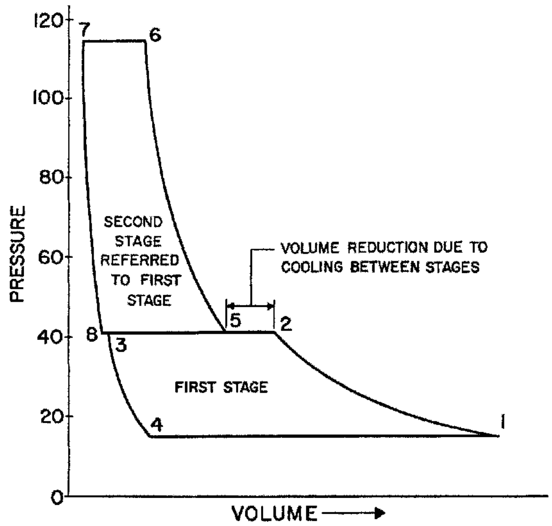

FIGURE 1.6 The p–V diagram for a two-stage compressor. *(Dresser-Rand Company, Painted Post, N.Y.)*

1.3 FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics states that energy cannot be created or destroyed during a process (such as compression and delivery of a gas), although it may change from one form of energy to another. In other words, whenever a quantity of one kind of energy disappears, an exactly equivalent total of other kinds of energy must be produced.

1.4 SECOND LAW OF THERMODYNAMICS

The second law of thermodynamics is more abstract and can be stated in several ways.

1. Heat cannot, of itself, pass from a colder to a hotter body.
2. Heat can be made to go from a body at lower temperature to one at higher temperature *only* if external work is done.
3. The available energy of the isolated system decreases in all real processes.
4. Heat or energy (or water), of itself, will flow only downhill.

Basically, these statements say that energy exists at various levels and is *available for use* only if it can move from a higher to a lower level.

In thermodynamics a measure of the *unavailability* of energy has been devised and is known as *entropy.* It is defined by the differential equation

$$dS = d\frac{Q}{T} \tag{1.1}$$

Note that entropy (as a measure of unavailability) increases as a system loses heat but remains constant when there is no gain or loss of heat (as in an adiabatic process).

1.5 IDEAL OR PERFECT GAS LAWS

An ideal or perfect gas is one to which the laws of Boyle, Charles, and Amonton apply. There are no truly perfect gases, but these laws are used and corrected by compressibility factors based on experimental data.

1.5.1 Boyle's Law

At constant temperature the volume of an ideal gas varies inversely with the pressure. In symbols:

$$\frac{V_2}{V_1} = \frac{p_1}{p_2} \tag{1.2}$$

$$p_2 V_2 = p_1 V_1 = \text{constant} \tag{1.3}$$

This is the *isothermal law*.

1.5.2 Charles' Law

The volume of an ideal gas at constant pressure varies directly as the absolute temperature:

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} \tag{1.4}$$

$$\frac{V_2}{T_2} = \frac{V_1}{T_1} = \text{constant} \tag{1.5}$$

1.5.3 Amonton's Law

At constant volume the pressure of an ideal gas will vary directly with the absolute temperature.

$$\frac{p_2}{p_1} = \frac{T_2}{T_1} \tag{1.6}$$

$$\frac{p_2}{T_2} = \frac{p_1}{T_1} = \text{constant} \tag{1.7}$$

1.5.4 Dalton's Law

Dalton's law states that the total pressure of a mixture of ideal gases is equal to the sum of the partial pressures of the constituent gases. The *partial pressure* is defined as the pressure each gas would exert if it alone occupied the volume of the mixture at the mixture temperature.

Dalton's law has been proven experimentally to be somewhat inaccurate, the total pressure often being higher than the sum of the partial pressures, particularly as pressures increase. However, for engineering purposes it is the best rule available and the error is minor. This can be expressed as follows, all pressures being at the same temperature and volume:

$$p = p_a + p_b + p_c + \cdots \tag{1.8}$$

1.5.5 Amagat's Law

Amagat's law is similar to Dalton's law but states that the volume of a mixture of ideal gases is equal to the sum of the partial volumes that the constituent gases would occupy if each existed alone at the *total* pressure and temperature of the mixture. As a formula this becomes

$$V = V_a + V_b + V_c + \cdots \tag{1.9}$$

Note: Dalton's and Amagat's laws are discussed further in Section 1.8.

1.5.6 Avogadro's Law

Avogadro's law states that equal volumes of all gases, under the same conditions of pressure and temperature, contain the same number of molecules. This law is very important and is applied in many compressor calculations. This is discussed further in Section 1.13.

1.5.7 Perfect Gas Formula

Starting with Charles' and Boyle's laws, it is possible to develop a formula for a given weight of gas:

$$pV = WR'T \tag{1.10}$$

where W is weight and R' is a specific constant for the gas involved. This is the perfect gas equation.

Going one step further, by making W in pounds equal to the molecular weight of the gas (1 mol), the formula becomes

$$pV = R_0T \tag{1.11}$$

This is very useful. R_0 is known as the *universal gas constant,* has a value of 1545, and is the same for all gases. Note, however, that R_0 is 1545 only when p is lb/ft^2; V is ft^3/lb-mol; and T is °R (°F + 460). When p is lb/in^2, R_0 becomes 10.729. The *specific gas constant* (R') for any gas can be obtained by dividing 1545 by the molecular weight.

1.6 VAPOR PRESSURE

As liquids change physically into a gas (as during a temperature rise), their molecules travel with greater velocity, and some break out of the liquid to form a vapor above the liquid. These molecules create a vapor pressure, which (at a specified temperature) is the only pressure at which a pure liquid and its vapor can exist in equilibrium.

If, in a closed liquid–vapor system, the volume is reduced at constant temperature, the pressure will increase imperceptibly until condensation of part of the vapor into liquid has lowered the pressure to the original vapor pressure corresponding to the temperature. Conversely, increasing the volume at constant temperature will reduce the pressure imperceptibly, and molecules will move from the liquid phase to the vapor phase until the original vapor pressure has been restored. Temperatures and vapor pressures for a given gas always move together.

The temperature corresponding to any given vapor pressure is obviously the *boiling point* of the liquid and also the *dew point* of the vapor. Addition of heat will cause the liquid to boil, and removal of heat will start condensation. The terms *saturation temperature, boiling point*, and *dew point* all mean the same physical temperature at a given vapor pressure. Their use depends on the context in which they appear.

Typical vapor pressure curves for common pure gases are shown in Appendix A. Tables of the properties of *saturated* steam show its temperature–vapor pressure relationship.

1.7 GAS AND VAPOR

By definition, a *gas* is a fluid having neither independent shape nor form, tending to expand indefinitely. A *vapor* is a gasified liquid or solid, a substance in gaseous form. These definitions are in general use today.

All *gases* can be liquefied under suitable pressure and temperature conditions and therefore could also be called *vapors*. The term *gas* is more generally used when conditions are such that a return to the liquid state (condensation) would be difficult within the scope of the operations being considered. However, a gas under such conditions is actually a superheated vapor.

The terms *gas* and *vapor* will be used rather interchangeably, with emphasis on closer approach to the liquid phase when using the word *vapor*.

1.8 PARTIAL PRESSURES

Vapor pressure created by one pure liquid will not affect the vapor pressure of a second pure liquid when the liquids are insoluble and nonreacting and the liquids and/or vapors are mixed within the same system. There is complete indifference on the part of each component to the existence of all others. The total vapor pressure for mixtures is the sum of the vapor pressures of the individual components. This is Dalton’s law, and each individual vapor has what is called a *partial pressure*, as differentiated from the total pressure of the mixture.

During compression of any gas other than a pure and dry gas, the principles of partial pressure are at work. This is true even in normal 100-psig air compression for power purposes, because there is always some water vapor mixed with the intake air and the compressor must handle both components. Actually, air is itself a mixture of a number of components, including oxygen, nitrogen, and argon, and its total pressure is the sum of the partial pressures of each component. However, because of the negligible variation in the composition of *dry* air throughout the world, it is considered and will hereafter be treated as a single gas with specific properties of its own.

After compression, partial pressures are used to determine moisture condensation and removal in intercoolers and aftercoolers. Partial pressures are also involved in many vacuum pump applications and are encountered widely in the compression of many mixtures.

Dalton's and Amagat's laws have been defined in Sections 1.5.4 and 1.5.5. See Eqs. (1.8) and (1.9), which apply here. Since water vapor is by far the most prevalent constituent involved in partial pressure problems in compressing gases, it is usually the only one considered in subsequent discussions.

In a mixture, when the dew-point temperature of any component is reached, the space occupied is said to be *saturated* by that component. A volume is sometimes specified as being *partially saturated* with water vapor at a certain temperature. This means that the vapor is actually superheated and the dew point is lower than the actual temperature. If the moles (see Section 1.13) of each component are known, the partial pressure of the component in question can be determined. Otherwise, it is customary to multiply the vapor pressure of the component at the existing mixture temperature by the relative humidity to obtain the partial pressure.

The terms *saturated gas* or *partially saturated gas* are incorrect and give the wrong impression. It is *not* the gas that is saturated with vapor; it is the volume or space occupied. The vapor and gas exist independently throughout the volume or space. Understanding of this true concept is helpful when working with partial pressures and gas mixtures.

Relative humidity is a term frequently used to represent the quantity of moisture present in a mixture, although it uses partial pressures in so doing. It is expressed as follows:

$$\text{RH}\ (\%) = \frac{\text{actual partial vapor pressure} \times 100}{\text{saturated vapor pressure at existing mixture temperature}}$$

$$= \frac{p_v \times 100}{p_s} \tag{1.12}$$

Relative humidity is usually considered only in connection with atmospheric air, but since it is unconcerned with the nature of any other components or the total mixture pressure, the term is applicable to vapor content in any problem, no matter what the conditions.

The saturated water vapor pressure at a given temperature is always known from steam tables or charts. It is the existing partial vapor pressure that is desired and is therefore calculable when the relative humidity is stated.

Specific humidity, used in calculations on certain types of compressors, is a totally different term. It is the ratio of the weight of water vapor to the weight of *dry* air and is usually expressed as pounds (or grains) of moisture per pound of dry air:

$$\text{SH} = \frac{W_v}{W_a} \tag{1.13}$$

or

$$\mathrm{SH} = \frac{0.622p_v}{p - p_v} = \frac{0.622p_v}{p_a} \tag{1.14}$$

where p_a is partial air pressure.

The *degree of saturation* denotes the actual relation between the weight of moisture existing in a space and the weight that would exist if the space were saturated:

$$\text{degree of saturation (\%)} = \frac{\text{SH actual} \times 100}{\text{SH saturated}} \tag{1.15}$$

$$= \mathrm{RH} \times \frac{p - p_s}{p - p_v} \tag{1.16}$$

Usually, p_s and p_v are quite small compared to p; therefore, the degree of saturation closely approximates the relative humidity. The latter term is commonly used in psychrometric work involving air–water vapor mixtures, whereas degree of saturation is applied mainly to gas–vapor mixtures having components other than air and water vapor.

The practical application of partial pressures in compression problems centers to a large degree around the determination of mixture volumes or weights to be handled at the intake of each stage of compression, the determination of mixture molecular weight, specific gravity, and the proportional or actual weight of components.

1.9 CRITICAL CONDITIONS

There is one temperature above which a gas will not liquefy with pressure increases no matter how great. This point is called the *critical temperature.* It is determined experimentally. The pressure required to compress and condense a gas at this critical temperature is called the *critical pressure.* The critical constants of many gases are given in Appendix A.

1.10 COMPRESSIBILITY

All gases deviate from the perfect or ideal gas laws to some degree, and in some cases the deviation is rather extreme. It is necessary that these deviations be taken into account in many compressor calculations to prevent cylinder volumes and driver sizes being sadly in error.

Compressibility is derived experimentally from data on the actual behavior of a particular gas under p–V–T changes. The compressibility factor Z is a multiplier in the basic formula. It becomes the ratio of the actual volume at a given p–T condition to the ideal volume at the same p–T condition.

The ideal gas equation (1.11) is therefore modified to

$$pV = ZR_0T \tag{1.17}$$

or

$$Z = \frac{pV}{R_0T} \tag{1.18}$$

In these equations, R_0 is 1545 and p is lb/ft^2.

A series of compressibility and temperature–entropy charts has been drafted to cover all gases on which reliable information could be found. These will be found in specialized texts or handbooks. In some cases, they represent consolidation and correlation of data from several sources, usually with a variance of less than 1% from the basic data. These charts may be considered authoritative.

Temperature–entropy charts are useful in the determination of theoretical discharge temperatures that are not always consistent with ideal gas laws. Discharge temperatures are required to obtain the compressibility factor at discharge conditions as involved in some calculations. These specific Z and T–S charts will provide the necessary correction factors for most compression problems involving the gases covered.

1.11 GENERALIZED COMPRESSIBILITY CHARTS

Because experimental data over complete ranges of temperature and pressure are not available for all gases, scientists have developed what are known as *generalized compressibility charts*. There are a number of these. One set has been selected as being suitable for screening calculations and is included in Appendix A.

These charts are based on what are called *reduced* conditions. Reduced pressure p_r is the ratio of the absolute pressure in lb/in^2 at a particular condition to the absolute critical pressure. Similarly, reduced temperature T_r is the ratio of the absolute temperature at the particular condition to the absolute critical temperature. The formulas are

$$p_r = \frac{p}{p_c} \tag{1.19}$$

$$T_r = \frac{T}{T_c} \tag{1.20}$$

It has been found that compressibility curves on the reduced basis for a large number of gases fall together with but small divergence. There are only a few gases that are too individualistic to be included.

Some charts show a reduced volume v_r' also, but this is really a pseudo (pretended)-reduced condition, obtained by use of the following formula (reduced volumes are not shown on the charts included here):

$$v_r' = \frac{v p_c}{R_0 T_c} \tag{1.21}$$

From this we can also write

$$v = \frac{v_r' R_0 T_c}{\mathrm{P}_c} \tag{1.22}$$

In these formulas, v and v_r' are the specific volumes of 1 mol of gas.

Critical pressures and temperatures for many gases are given in Appendix A.

1.12 GAS MIXTURES

Mixtures can be considered as equivalent ideal gases. Although this is not strictly true, it is satisfactory for the present purposes. Many mixtures handled by compressors contain from 2 to 10 separate components. It is necessary to determine as closely as possible many properties of these as *equivalent gases.* Chief among these properties are:

- Specific volume
- Density
- Volume and mole percent
- Molecular weight
- Specific gravity
- Partial pressure
- Ratio of specific heats (k)
- Pseudo-reduced pressure
- Pseudo-reduced temperature
- Compressibility
- Gas constant
- Specific heats

1.13 THE MOLE

The *mole* is particularly useful when working with gas mixtures. It is based on Avogadro's law: that equal volumes of gases at given pT conditions contain equal numbers of molecules. Since this is so, the *weight* of these equal volumes will be proportional to their molecular weights.

The volume of 1 mol at any desired condition can be found by the use of the perfect gas law:

$$pV = R_0T \qquad \text{or} \qquad pV = 1545T \tag{1.23}$$

Choosing standard pressure and temperature (SPT) conditions, we solve for V in the previous formula (p is lb/ft^2 and T is °R). This turns out to be 379.4 ft^3. For simplicity, use 379 ft^3/mol. To repeat, this is the volume of a weight (expressed in pounds) of any gas at 14.696 psia and 60°F—the weight being the same number as the molecular weight.

Thus, a mole of hydrogen occupies a volume of 379 ft^3 at standard conditions and weighs 2.016 lb. A mole of air occupies 379 ft^3 at the same conditions but weighs 28.97 lb. A mole of isobutane, still 379 ft^3, weighs 58.12 lb. This, of course, assumes that they act as perfect or ideal gases, which most of them do at or near standard conditions (SPT): 14.696 psia and 60°F. Most mole calculations involve these or similar conditions. Note, however, that a mole is a *weight* of gas. It is *not* a volume.

Despite the deviation from a perfect gas sometimes being in question, the following methods of obtaining mixture pseudo properties are of great value, and in some cases are the only approach.

1.14 SPECIFIC VOLUME AND DENSITY

Since the volume and the weight of a mole of any gas is known from the defined relations, it follows that the specific volume in ft^3/lb or density in lb/ft^3 is obtained by simple division.

Gas	Specific Volume (ft^3/mol)	lb/mol and Molecular Weight	Specific Volume (ft^3/lb)	Density (lb/ft^3)
Hydrogen	379	2.016	188.3	0.00531
Air	379	28.97	13.1	0.0763
Isobutane	379	58.12	6.51	0.153

Note that these data are on the basis of perfect gas laws. Some gases—isobutane is one—deviate even at SPT conditions. The actual figures on isobutane, for example, are 6.339 ft^3/lb and 0.1578 lb/ft^3.

1.15 VOLUME PERCENT OF CONSTITUENTS

Mole percent is the ratio of the number of moles of one constituent to the total number of moles of mixture. Mole percent also happens to be percent by volume. This statement should be questioned since a mole is defined as a weight. Look at the following table for proof. The gas analysis in these and following tables is that of a typical raw ammonia synthesis gas.

Gas	Mol %	Mol/mol of Mixture	Volume (SPT) of 1 mol	Volume/mol of Mixture	Vol %
H_2	61.4	0.614	379	232.7	61.4
N_2	19.7	0.197	379	74.7	19.7
CO_2	17.5	0.175	379	66.3	17.5
CO	1.4	0.014	379	5.3	1.4
	100.0	1.000		379.0	

1.16 MOLECULAR WEIGHT OF A MIXTURE

The average molecular weight of the mixture is often needed. It is obtained by multiplying the molecular weight of each component by its mole fraction (mol %/100) and adding these values as follows:

Gas	Mol % or Vol %	Mol. Wt.	Proportional Mol. Wt.
H_2	61.4	2	1.228
N_2	19.7	28	5.516
CO_2	17.5	44	7.700
CO	1.4	28	0.392
	100.0		14.84

Therefore, the average (or pseudo) molecular weight of the mixture is 14.84.

1.17 SPECIFIC GRAVITY AND PARTIAL PRESSURE

Normally, specific gravity for gases is a ratio of the lb/ft^3 of the gas involved to the lb/ft^3 of air, both at SPT conditions. Considering a mole of each gas, the volumes are the same and the weight of each volume is the same as the molecular weight. Therefore, specific gravity is calculated as the ratio of these molecular weights and becomes (for the previous example) 14.84 divided by 28.97, or 0.512.

It can be stated that the fraction of the total pressure in a gas mixture due to a given component is equal to the fraction which that component represents of the total moles of gas present:

$$p_a = \frac{pN_a}{N} \quad \text{and} \quad p_b = \frac{pN_b}{N} \quad \text{and} \quad p_c = \frac{pN_c}{N} \tag{1.24}$$

Thus, in a mixture of 15 mol at 15 psia total pressure containing 2 mol of hydrogen, the partial pressure of the hydrogen would be 2/15 of 15 psia, or 2 psia. Volume fractions, if available, may be used in place of mole fractions here.

1.18 RATIO OF SPECIFIC HEATS

The value of k enters into many calculations. A definite relationship exists between the specific heat at constant volume and the specific heat at constant pressure. If we take a mole of gas and determine its heat capacity:

$$Mc_p = Mc_v + 1.99 \tag{1.25}$$

$$Mc_v = Mc_p - 1.99 \tag{1.26}$$

In these formulas, M is the weight of a mole of gas (the molecular weight). These are easily resolved into

$$k = \frac{Mc_p}{Mc_v} = \frac{Mc_p}{Mc_p - 1.99} \tag{1.27}$$

Remembering the unit of specific heat as Btu/lb-°F temperature rise, we can calculate the heat required to increase the temperature of each component gas by 1°F and add them to get the total for the mixture. Mc_p is the heat requirement for 1 mol. For compressor work, it is usual to use this molar heat capacity at 150°F, which is considered an average temperature. A calculation table follows:

Gas	Mol %	Mol Gas/Mol Mixture	Mc_p at 150°F of Component	Product
H_2	61.4	0.614	6.94	4.26
N_2	19.7	0.197	6.98	1.38
CO_2	17.5	0.175	9.37	1.64
CO	1.4	0.014	6.97	0.10
	100.0	1.000		7.38

Note: For convenience, the molar heat capacity at 150°F (Mc_p) is given in Appendix A for most gases.

The molar specific heat (Mc_p) of the mixture is therefore 7.38. Entering this in the formula yields

$$k = \frac{7.38}{7.38 - 1.99} = 1.369 \text{ (say, 1.37)} \tag{1.28}$$

1.19 PSEUDO-CRITICAL CONDITIONS AND COMPRESSIBILITY

Mention has been made of reduced pressure and reduced temperature under the discussion of compressibility. Generalized compressibility curves on this basis are given in Appendix A. They are also applicable to mixtures—for approximations, at least. For a more rigorous treatment, texts such as Ried, et al. [1] should be consulted.

It is necessary to figure mixture pseudo-critical pressure and temperature conditions to be used in calculating the pseudo-reduced conditions to be used in entering the charts. Pressures and temperatures must be in absolute values.

Gas	Mol %	Individual Critical Temperature (°R)	Pseudo T_c (°R)	Individual Critical Pressure (psiA)	Pseudo p_c (psia)
H_2^a	61.4	83	51.0	327	201.0
N_2	19.7	227	44.7	492	96.9
CO_2	17.5	548	95.9	1073	187.8
CO	1.4	242	3.4	507	7.1
Mixture pseudo-criticals			195		493

[a] Must use effective critical conditions (see Appendix A.)

Using these values, the pseudo-reduced conditions can be calculated and probable Z factors obtained from generalized charts.

1.20 WEIGHT-BASIS ITEMS

To certain gas properties of a mixture, each component contributes a share of its own property in proportion to its fraction of the total *weight*. Thus, the following are obtained, the weight factors being fractions of the whole:

$$R' = \frac{W_a R'_a + W_b R'_b + W_c R'_c + \cdots}{W} \tag{1.29}$$

$$c_p = \frac{W_a c_{pa} + W_b c_{pb} + W_c c_{pc} + \cdots}{W} \tag{1.30}$$

$$c_v = \frac{W_a c_{va} + W_b c_{cb} + W_c c_{rc} + \cdots}{W} \tag{1.31}$$

1.21 COMPRESSION CYCLES

Two theoretical compression cycles are applicable to positive displacement compressors. Although neither cycle is commercially attainable, they are used as a basis for calculations and comparisons.

Isothermal compression occurs when the temperature is kept constant as the pressure increases. This requires continuous removal of the heat of compression. Compression follows the formula

$$p_1V_1 = p_2V_2 = \text{constant} \tag{1.32}$$

Near-adiabatic (*isentropic*) *compression* is obtained when there is no heat added to or removed from a gas during compression. Compression follows the formula

$$p_1V_1^k = p_2V_2^k \tag{1.33}$$

where k is the ratio of the specific heats.

Figure 1.7 shows the theoretical no-clearance isothermal and adiabatic cycles on a pV basis for a compression ratio of 4. Area ADEF represents the work required when operating on the isothermal basis, and ABEF, the work required on the adiabatic basis. Obviously, the isothermal area is considerably less than the adiabatic and would be the cycle for greatest compression economy. However, the isothermal cycle is not commercially approachable, although compressors are usually designed for as much heat removal as possible.

Similarly, adiabatic compression is never obtained exactly, since some heat is always rejected or added. Actual compression therefore takes place along a *polytropic cycle*, where the relationship is

$$p_1V_1^n = p_2V_2^n \tag{1.34}$$

The exponent n is determined experimentally for a given type of machine and may be lower or higher than the adiabatic exponent k. In positive displacement compressors, n is usually

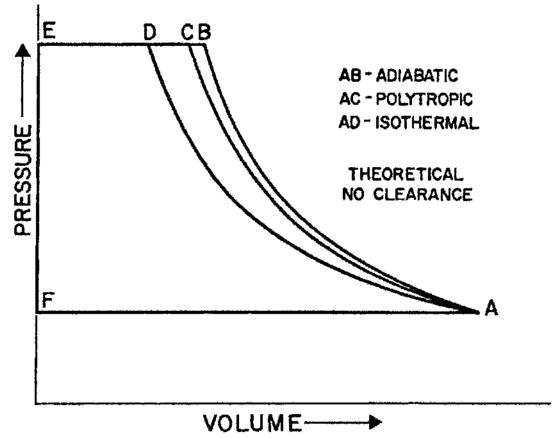

FIGURE 1.7 A p–V diagram illustrating theoretical compression cycles. *(Dresser-Rand Company, Painted Post, N.Y.)*

less than k. Figure 1.7 shows a typical polytropic compression curve for a reciprocating water-jacketed compressor cylinder.

Thermodynamically, it should be noted that the isentropic or adiabatic process is reversible, whereas the polytropic process is irreversible. Also, all compressors operate on a *steady-flow* basis.

Either n or $(n - 1)/n$ can also be experimentally calculated from test data if inlet and discharge pressures and temperatures are known. The following formula may be used:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n} = r^{(n-1)/n} \tag{1.35}$$

This formula can also be used to estimate discharge temperatures when n or $(n - 1)/n$ is known.

It is obvious that k and n can have quite different values. In certain engineering circles, there has been a tendency to use these symbols interchangeably to represent the ratio of specific heats. This is incorrect; they should be differentiated carefully.

1.22 POWER REQUIREMENT

The power requirement of any compressor is the prime basis for sizing the driver and for selection and design of compressor components. The actual power requirement is related to a theoretical cycle through a *compression efficiency,* which has been determined by test on prior machines. Compression efficiency is the ratio of the theoretical to the actual gas horsepower and, as used by the industry, does not include mechanical friction losses. These are added later either through the use of a mechanical efficiency or by adding actual mechanical losses previously determined. Positive displacement compressors commonly use mechanical efficiencies ranging from 88 to 95%, depending on the size and type of unit.

Historically, the isothermal cycle was the basis used for many years. It is used today in only a few cases. Positive displacement machines are now compared to the isentropic or adiabatic cycle, which more nearly represents what actually occurs in the compressor. In calculating horsepower, the compressibility factor Z must be considered since its influence is considerable with many gases, particularly at high pressure.

An inlet volume basis is universal with positive displacement compressors. It is important to differentiate between an inlet volume on a perfect gas basis (V_{p1}) and one on a real gas basis (V_{r1}). Volumes are at inlet pressure and temperature (p_1 and T_1):

$$V_{r1} = V_{p1} Z_1$$

The basic theoretical adiabatic single-stage horsepower formula is

$$P_T(\text{ad}) = \frac{p_1 V_{r1}}{229} \frac{k}{k-1} (r^{(k-1)/k} - 1) \frac{Z_1 + Z_2}{2Z_1} \tag{1.36}$$

This represents the area of a theoretical adiabatic p–V diagram for the volume per minute (V_1) being handled.

A frequently used basis for V_1 is 100 cfm (real) at inlet conditions, in which case the formula becomes

$$\frac{P_T(\text{ad})}{100} = \frac{P_1}{2.29}\frac{k}{k-1}(r^{(k-1)/k} - 1)\frac{Z_1 + Z_2}{2Z_1} \tag{1.37}$$

Another form current in the industry is the basis for frequently used charts. In this, a volume of 1 million ft^3/day (MMcfd) is used. *In this case only,* V_1 is measured as *perfect gas* at *14.4 psia* and intake temperature, and the actual compressor capacity must be referred to these conditions before computing the final horsepower.

$$\frac{P_T(\text{ad})}{\text{MMcfd}} = 43.67\frac{k}{k-1}(r^{(k-1)/k} - 1)\frac{Z_1 + Z_2}{2} \tag{1.38}$$

Since the isothermal cycle is based on no temperature change during compression, heat is removed continuously as generated, and there is theoretically no gain by multiple staging. Therefore, Eq. (1.39) holds for any number of stages as long as r is the overall or total compression ratio:

$$P_T(\text{iso}) = \frac{p_1 V_{r1} \ln r}{229}\frac{Z_1 + Z_2}{2Z_1} \tag{1.39}$$

1.23 COMPRESSIBILITY CORRECTION

In the preceding equations a correction is indicated for deviation from the perfect gas laws—the compressibility. This involves the determination of compressibility at both intake and discharge conditions. Intake pressure and temperature are known, and compressibility at these conditions can be obtained directly from specific gas charts or by the reduced condition method using generalized charts. To obtain Z at discharge conditions, it is necessary to determine the discharge temperature. The discharge pressure is known.

On the adiabatic cycle as applied to positive displacement units, it is customary to use the *theoretical* discharge temperature in calculations. In an actual compressor, many factors are acting to cause variation from the theoretical, but *on average,* the theoretical temperature is closely approached, and any error introduced is slight.

Adiabatic compression is *isentropic* (i.e., the entropy remains constant). If temperature–entropy diagrams are available for the gas involved, the theoretical discharge temperature can be read directly. Otherwise, it is necessary to calculate it using the following relationships:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = r^{(k-1)/k} \tag{1.40}$$

Note that all pressures and temperatures are absolute.

Equations (1.36) through (1.39) are theoretical and are not affected by gas characteristics such as molecular weight, specific gravity, and actual density at operating conditions.

These all have an effect on actual power requirements, however, and proper allowances are made by designers.

1.24 MULTIPLE STAGING

All *basic* compressor elements, regardless of type, have certain limiting operating conditions. Basic elements are single stage (i.e., the compression and delivery of gas is accomplished in a single element) or a group of elements are arranged in parallel. The most important limitations include the following:

- Discharge temperature
- Pressure differential
- Effect of clearance (ties in with compression ratio)
- Desirability of saving power

There are reasons for multiple staging other than these, but they are largely for the designer of the specific unit to keep in mind. No ready reference rules can be given. When any limitation is involved, it becomes necessary to multiple-stage the compression process (i.e., do it in two or more steps). Each step will use at least one *basic element* designed to operate in series with the other elements of the machine.

A reciprocating compressor usually requires a separate cylinder for each stage with intercooling of the gas between stages. Figure 1.8 shows the *p–V* combined diagram of a two-stage 100-psig air compressor. Further stages are added in the same manner. In a reciprocating unit, all stages are commonly combined into one unit assembly.

It was noted previously that the isothermal cycle (constant temperature) is the more economical of power. Cooling the gas after partial compression to a temperature equal to the

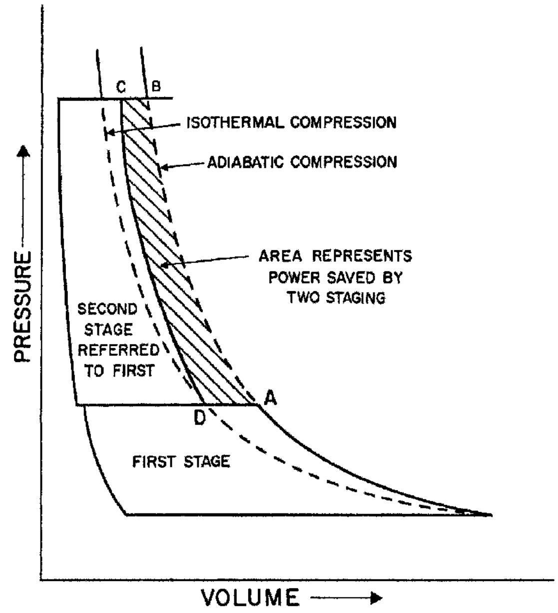

FIGURE 1.8 Combined *p–V* diagram for a two-stage air compressor. *(Dresser-Rand Company, Painted Post, N.Y.)*

original intake temperature (back to the isothermal) obviously should reduce the power required in the second stage. Area ABCD represents the work saved over single-stage adiabatic compression in this particular case.

For minimum power *with perfect intercooling* between stages, there is a theoretically best relation between the intake pressures of succeeding stages. This is obtained by making the ratio of compression the same in each stage and assumes the intake temperature to be the same in all stages. The formula used is based on the *overall* compression ratio

$$r_s = \sqrt[s]{r_t} \tag{1.41}$$

where r_s = compression ratio per stage
s = number of stages
r_t = overall compression ratio ($p_{\text{final}}/p_{\text{initial}}$)

For example:

$$\begin{aligned} &\text{two-stage:} && r_s = \sqrt[2]{r_t} \\ &\text{three-stage:} && r_s = \sqrt[3]{r_t} \\ &\text{four-stage:} && r_s = \sqrt[4]{r_t} \end{aligned}$$

Commercial displacement compressors are, as a rule, initially sized on the preceding basis. The final machine, however, usually operates at compression ratios varying slightly from these to allow for other factors that the designer must consider. Each stage is figured as a separate compressor, the capacity (V_1) of each stage being calculated separately from the first-stage real intake volume, and corrected to the actual pressure and temperature conditions existing at the higher-stage cylinder inlet and also for any change in the moisture content if there is condensation between stages in an intercooler. The theoretical power per stage can then be calculated and the total horsepower obtained.

On the basis of perfect intercooling and equal compression ratios per stage, Eqs. (1.36) through (1.38) can be altered to obtain *total* theoretical power by multiplying the first term by the number of stages s and dividing the exponent of r by s. The compression ratio r must be the total ratio. However, since compression ratios seldom are equal and perfect intercooling is seldom attained, it is believed that the best general method of figuring is to use one stage at a time.

1.25 VOLUME REFERENCES

Since the most generally required quantities are original inlet volume and inlet volume to subsequent stages (both on a per-minute basis), a summary of equations follows in which the word *dry* means that there is no water vapor in the quantity of gas or gas mixture involved. From scfm (cfm measured at 14.7 psia, 60°F, dry),

$$V_1 = \text{scfm}\left(\frac{14.7}{p_1}\right)\frac{T_1}{520}Z_1 \tag{1.42}$$

From weight flow (W lb/min, dry),

$$V_1 = \frac{W(1545)T_1}{144p_1M} Z_1 \qquad (1.43)$$

From mole flow (*N* mol/min, dry),

$$V_1 = \frac{N(379)(14.7)T_1}{p_1(520)} Z_1 \qquad (1.44)$$

From cfm measured at conditions other than those at intake cfm_g *at* p_g, T_g, Z_g, *dry*:

$$V_1 = \text{cfm}_g \times \frac{p_g}{p_1}\frac{T_1}{T_g}\frac{Z_1}{Z_g} \qquad (1.45)$$

In all the preceding, pressure is lb/in^2 absolute.

If water vapor is a component in the gas analysis and the total analysis percentage amounts to 100, the preceding equations may be applied to the wet gas. Use the proper value for M in Eq. (1.43), however. Often, water vapor is segregated, and the space it occupies must be included separately. This is a partial pressure problem (see Section 1.8). Multiplying any of the preceding volume equations by the following will apply the correction:

$$\frac{p_1}{p_1 - p_v} \qquad (1.46)$$

where p_v is the actual vapor pressure of the contained moisture.

1.26 CYLINDER CLEARANCE AND VOLUMETRIC EFFICIENCY

Cylinder clearance cannot be eliminated completely. *Normal* clearance will vary approximately between 4 and 16% for most standard cylinders. There are special low-compression-ratio cylinders where normal clearance will be much greater. Normal clearance does not include clearance volume that may have been added for other purposes, such as capacity control.

Although the amount of clearance in a given cylinder is of little importance to the average user (guarantees being made on actually delivered capacity), its effect on capacity should be understood because of the wide application of a variation in clearance volume for capacity control and other purposes. Normal clearance variations have no effect on power requirements.

When a piston has completed the compression and delivery stroke and is ready to reverse its movement, gas at discharge pressure is trapped in the clearance space. This gas expands on the return stroke until its pressure is sufficiently below intake pressure to open the suction valves. The effect of this reexpansion on the quantity of fresh gas drawn in is shown on a p–V diagram (Fig. 1.9). The actual capacity is materially affected.

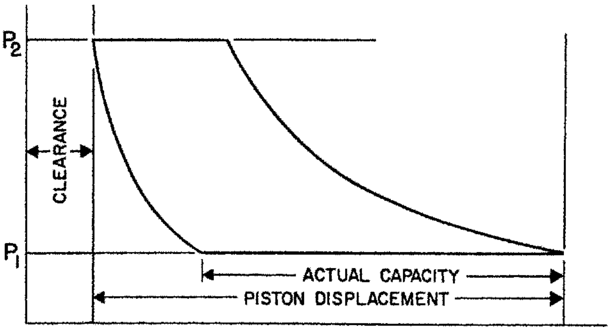

FIGURE 1.9 Work done on a volume of gas trapped in cylinder clearances (clearance volume) represents an inefficiency. *(Dresser-Rand Company, Painted Post, N.Y.)*

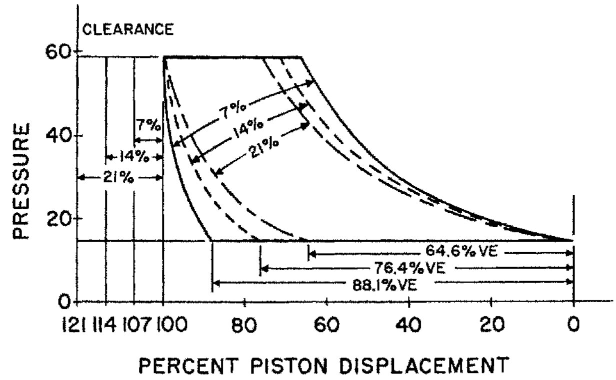

FIGURE 1.10 Theoretical *p–V* diagrams based on a compression ratio of 4.0, *k* of 1.40, and clearances of 7, 14, and 21% *(Dresser-Rand Company, Painted Post, N.Y.)*

The theoretical formula for volumetric efficiency as a percentage is

$$\eta_v = 100 - C(r^{1/k} - 1) \tag{1.47}$$

As a practical matter, there are factors that modify this, and an accepted formula for rough estimates is

$$\eta_v = 100 - C(r^{1/k} - 1) - L \tag{1.48}$$

Here, the term *L* is introduced to allow for the effect of variables such as internal leakage, gas friction, pressure drop through valves, and inlet gas preheating. The term *L* is difficult to generalize, but it might be 5% for a moderate-pressure oil-lubricated air compressor. A higher value of *L* will be necessary with a light gas than with a heavy gas because of increased leakage.

Inspection of the equations shows that the *VE decreases* (1) as the clearance *increases,* (2) as the compression ratio *increases,* and (3) as *k decreases.*

Figure 1.10 shows a series of theoretical *p–V* diagrams based on an *r* value of 4.0, a *k* value of 1.40, and clearances of 7, 14, and 21%. The effect of clearance is clearly indicated. Rather wide clearance ranges have been used, for illustrative purposes.

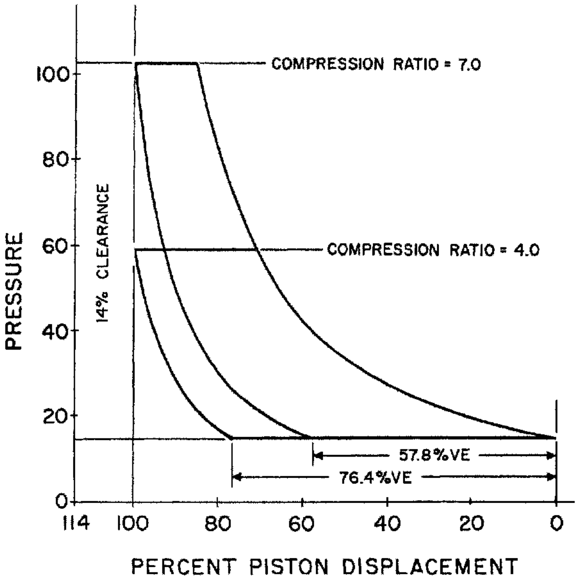

FIGURE 1.11 Effect of clearance at moderate- and high-compression-ratio conditions. A *p–V* diagram for a ratio of 7 is superimposed on a diagram for a ratio of 4, all else being the same. *(Dresser-Rand Company, Painted Post, N.Y.)*

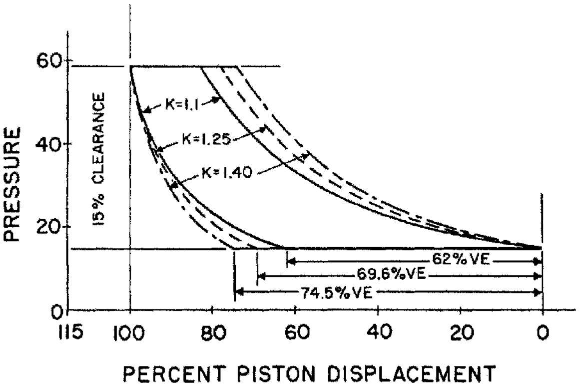

FIGURE 1.12 Effect of *k* on volumetric efficiency. The clearance is high, for illustrative purposes. *(Dresser-Rand Company, Painted Post, N.Y.)*

Figure 1.11 illustrates the effect of clearance at moderate- and high-compression-ratio conditions. A *p–V* diagram for a ratio of 7 is superimposed on a diagram for a ratio of 4, all else being the same. A relatively high clearance value (14%) is used for illustrative purposes. The clearance for any commercial compressor designed for a ratio of 7 would be much less than 14%.

Figure 1.12 illustrates the effect of *k* on volumetric efficiency. The clearance is high for illustrative purposes. Clearance obviously concerns designers more at the higher compression ratios and when handling gases with low specific heat ratios, although they will always endeavor to maintain clearance at the lowest value consistent with adequate valving and running clearances.

1.27 CYLINDER CLEARANCE AND COMPRESSION EFFICIENCY

Just as clearance in a cylinder has predominant control over the volumetric efficiency (VE), so does the valve area in a cylinder have predominant control over the compression efficiency (CE). However, to obtain low clearance and a high VE value, the designer finds it necessary to limit the size and number of valves. This, however, may tend to lower the efficiency of compression and raise the horsepower. The designer must therefore evaluate both factors and arrive at a compromise, quite a common engineering procedure.

As a general rule, high VE and high CE (low power requirement) do not go together; one cannot attain both. There are, however, four rough divisions the designer uses, the type of application determining to a considerable degree whether one or the other factor takes precedence or whether they are balanced. They might be classified as follows:

	Compression Ratio	More Important Factor
Very high	10–30 (vacuum pumps)	Clearance
High	8–10 max.	Clearance principally, valving somewhat
Moderate	5 max.	Balanced
Low	2 or less	Valving

REFERENCE

1. Ried, Prausnitz, and Poling, *The Properties of Gases and Liquids*, 4th ed., McGraw-Hill, NY, 1988.