11

SIMPLIFIED EQUATIONS FOR DETERMINING THE PERFORMANCE OF DYNAMIC COMPRESSORS*

The following data comprise the fundamental equations that are used in the determination of brake horsepower, operating speeds, and discharge temperature of centrifugal and axial gas compressors.

11.1 NONOVERLOADING CHARACTERISTICS OF CENTRIFUGAL COMPRESSORS

Impellers with backward-leaning vanes have the characteristic that at constant speed, the discharge pressure in the head decreases gradually with increasing capacity. Thus, at rated suction temperature and pressure, it is not possible to overload a properly selected prime mover since both the head and the brake horsepower will decrease appreciably as the capacity increases above 120% of the rated capacity.

11.2 STABILITY

Stability is defined in conjunction with a *surge point*. Dynamic compressors surge, or undergo a reversal of flow direction, when the gas throughput drops below a certain value that is defined uniquely by compressor geometry, operating conditions, gas properties, and other variables. This flow reversal usually takes place at or near the impeller tip; it can cause process upsets and/or serious mechanical damage to compressor internals. It is discussed further in Chapter 16.

* Contributed by Dresser-Rand Company, Olean, N.Y., except as noted.

A Practical Guide to Compressor Technology, Second Edition, By Heinz P. Bloch

The percent of change in capacity between the rated capacity and the surge point, at rated head, is measured as the stability of the centrifugal compressors (Fig. 11.1). This value will vary from approximately 70% for compressors developing very low pressure ratios to as low as 30% for compressors developing very high ratios. In the initial design of a compressor,

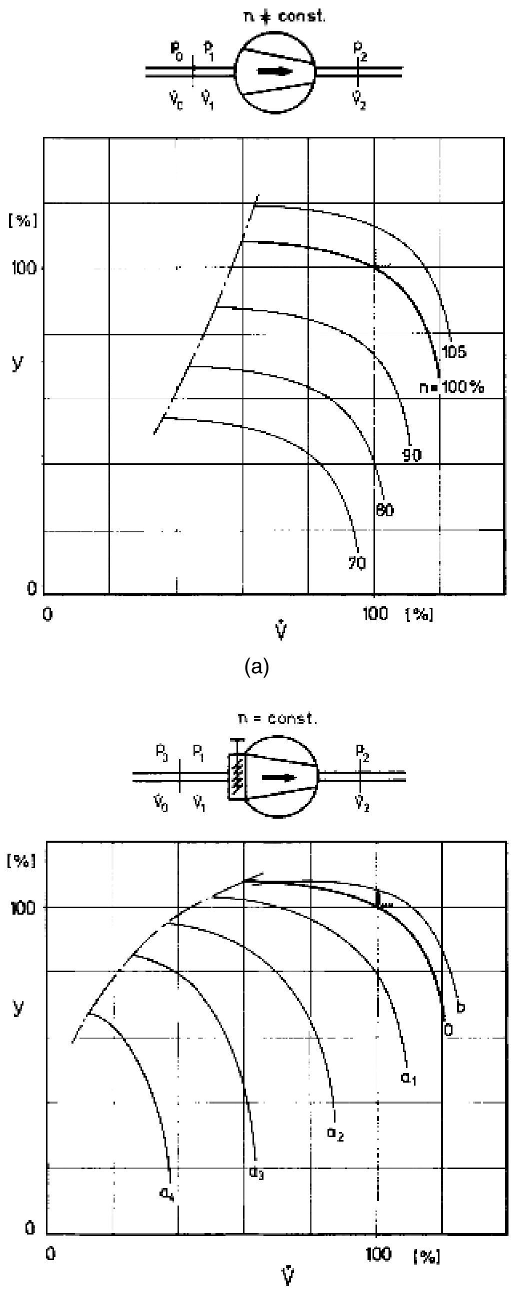
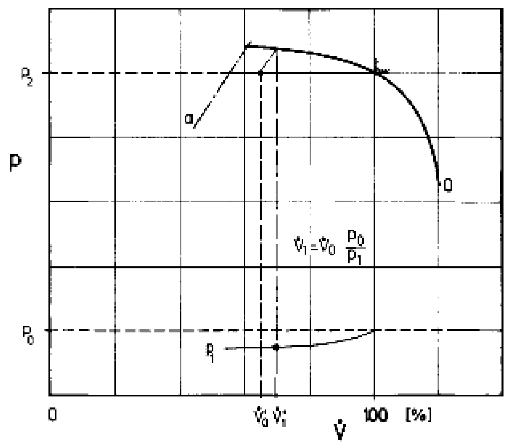


FIGURE 11.1 Performance of dynamic compressors: (*a*) variable-speed; (*b*) variable inlet guide vanes; (*c*) suction valve throttling. *(Sulzer, Ltd., Winterthur, Switzerland)*



(c)

FIGURE 11.1 (*continued*)

provision can be made for high stability at slightly reduced efficiency if it is likely that partial loads will be of long duration. When the design load is sustained most of the time, the efficiency can be improved at the expense of stability.

Figure 11.1 also shows how the performance of dynamic compressors is influenced by different control methods: (1) operation at speeds ranging from 70 to 105% of original design, (2) operation at constant speed but with different guide vane settings, and (3) operation at constant speed and suction valve throttling. In each case, the vertical axis represents either head or pressure developed, while flow (in cfm or m^3/h) is represented on the horizontal axis.

11.3 SPEED CHANGE

The large difference in head with small changes in speed is illustrated by the head capacity curve (Fig. 11.1*a*). This shows that centrifugal compressor prime movers are sometimes designed for operation between 70 and 105% of the rated speed. Operation without speed change results in maintaining a head vs. flow relationship as described by the performance curve of Fig. 11.2. Note that operation to the left of the surge limit and in the choke flow or *stonewall region* is not feasible.

11.4 COMPRESSOR DRIVE

The type of prime mover that is used for centrifugal compressors will be determined in most cases by the economics of the application. There are four different classes of prime movers that are considered most suited for centrifugal compressors:

- Steam turbine
- Electric motor

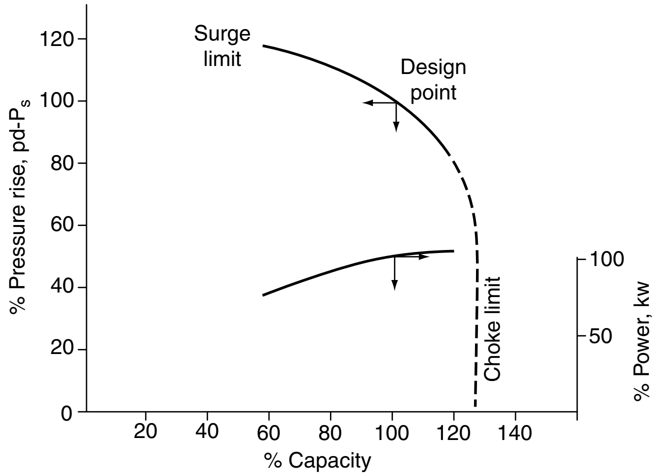


FIGURE 11.2 Head capacity curve for a centrifugal compressor. *(Dresser-Rand Company, Olean, N.Y.)*

- Expansion turbine (expander)
- Combustion gas turbine

Most centrifugal compressors have been built for drive by the four types mentioned, illustrating the flexibility with which they may be applied. Driver selection is determined primarily by the following factors:

- Water requirements (steam consumption)
- Operating speeds
- Process control
- Process steam supply
- Fuel or energy costs
- Reliability

Adjustable inlet guide vanes or suction throttling are available for constant-speed motor drive. The electric motor may also be used for variable-speed operation by the use of hydraulic couplings or other means.

11.5 CALCULATIONS

The three items usually to be determined in centrifugal compressor calculations are:

- Shaft horsepower
- Operating speed
- Discharge temperature

Determination of horsepower and speed is predicated on the calculation of the head required for the compression. Head, which actually represents the work being done per pound of fluid being handled, is expressed in terms of ft-lb/lb or N · m/kg, just as for a liquid pump, and is fundamentally defined by the following:

$$H = k_1 \int V\, dP \tag{11.1}$$

where H = head, ft (m)
V = specific volume, ft^3/lb (m^3/kg)
P = pressure, psia (bar, absolute)
k_1 = different constants, for English or metric conversions and expressions

For a liquid pump, where the specific volume or density is constant, Eq. (11.1) is readily integrated to

$$H = k_1 V(P_2 - P_1) = \frac{k_1(P_2 - P_1)}{\rho} \tag{11.2}$$

where ρ is the density in lb/ft^3 or kg/m^3.

For a centrifugal compressor, where the specific volume is a variable, a somewhat more complex relation is obtained. If it is assumed that the compression is polytropic and may be represented by the equation

$$PV^n = \text{constant}$$

Eq. (11.1) may be integrated and rearranged to the familiar form

$$H = \frac{k_1 P_1 V_1}{(n-1)/n}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] \tag{11.3}$$

where n is the polytropic exponent of compression.

Equation (11.3) may alternatively be expressed in the form

$$H = \frac{ZRT_1}{(n-1)/n}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] \tag{11.4}$$

where R = gas constant = 1545/MW or 8314/MW
T_1 = suction temperature, °R or K
Z = average compressibility

For ease in calculation, Eqs. (11.3) and (11.4) may be expressed in the form

$$H = k_1 P_1 V_1 \beta = ZRT_1\beta \tag{11.5}$$

where $\beta = [(P_2/P_1)^M - 1]/M$
$M = (n - 1)/n$

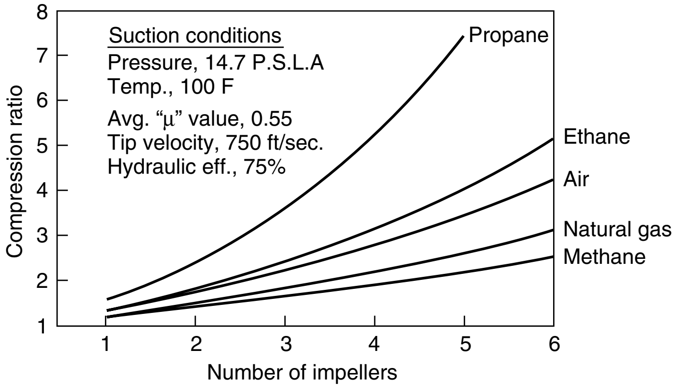


FIGURE 11.3 Compression ratio vs. number of impellers. *(Dresser-Rand Company, Olean, N.Y.)*

From Eqs. (11.4) and (11.5), it is therefore apparent that the head, and hence the horsepower, for a given compression vary directly with the absolute suction temperature and inversely with the molecular weight of the gas being handled. Since there is a limit to the amount of head that a single impeller will develop, as will subsequently be demonstrated, it follows that gases having a high molecular weight will require fewer impellers (i.e., stages) than will gases having a low molecular weight when being compressed through the same ratio. This is indicated in Fig. 11.3 for several common gases.

For perfect gas compression, the compressibility factor is unity. For real gas compression, this factor deviates from unity. In those instances where the amount of this deviation is not large (i.e., where the average compressibility factor varies between 0.95 and 1.02 or where it remains fairly constant over the range of compression), an average value of the compressibility factor may be used in Eq. (11.4) with negligible error. In other instances, where the compressibility factor is subject to larger variation over the range of compression, the head may be approximated from the following relation:

$$H = k_2[(P_1V_1) + (P_2V_2)] \log_{10} \frac{P_2}{P_1} \tag{11.6}$$

where k_2 represents different constants in the English and metric systems of measurement.

Equation (11.6) is not strictly correct. It is based on the assumption that the log mean value of PV equals the arithmetic mean. This will result in an error of 1.2% for high compression ratios and is based only on the assumption that the compression is polytropic and can be represented by a single exponent (n).

To be correct, the following formula should be used:

$$H = k_1 \log_{10} \frac{P_2}{P_1} \frac{P_2V_2 - P_1V_1}{\log_{10}(P_2V_2/P_1V_1)} \tag{11.7}$$

Equation (11.6) is of particular utility for hydrocarbon gases at moderate or high pressures and/or low temperatures.

It is to be noted that successful use of Eqs. (11.3) and (11.4) is dependent on the determination of the polytropic exponent. This may be readily obtained from the following equations, which follow from the definition of hydraulic efficiency:

$$\eta = -\frac{\int V\,dP}{\Delta h} = \frac{\dfrac{P_1V_1}{(n-1)/n}\left[\left(\dfrac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]}{\dfrac{P_1V_1}{(K-1)/K}\left[\left(\dfrac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]} \tag{11.8}$$

$$\eta = \frac{(K-1)/K}{(n-1)/n} \tag{11.9}$$

where η = hydraulic efficiency
Δh = change in enthalpy, Btu/lb or kJ/kg
K = isentropic exponent (c_p/c_v)

The hydraulic efficiency is established by tests and is generally a function of the capacity at suction conditions to the compressor.

The head that a centrifugal compressor stage consisting of an impeller and diffuser will develop may be related to the peripheral velocity by the following:

$$H = \mu \frac{u^2}{g} \tag{11.10}$$

where μ = pressure coefficient
u = peripheral velocity, ft/s or m/s
g = gravitational constant, 32.2 ft/s^2 or 9.81 m/s^2

The value of the pressure coefficient referred to previously is a characteristic of the stage design. An average value for one stage of a multistage centrifugal compressor is 0.55. If a peripheral velocity of 770 ft/s (235 m/s) is assumed, it can be seen that the head per stage is approximately 10,000 ft (3050 m). This permits ready approximation of the number of stages required to develop the head corresponding to the particular compression process.

The power required for the compression of a gas may be calculated from the following:

$$\text{ghp} = \frac{W\Delta h}{33{,}000} \tag{11.11}$$

or

$$\text{kW} = \frac{mH}{3600}$$

where ghp = gas horsepower
W = gas flow, lb/min
kW = gas power, kW
m = mass flow, kg/h
H = differential head, m

From Eq. (11.8), however,

$$\Delta h = \frac{\int V\,dP}{\eta} = \frac{H}{\eta} \tag{11.12}$$

Therefore,

$$\text{ghp} = \frac{WH}{33{,}000\eta}$$

and

$$\text{kW} = \frac{mH_p}{3600\eta}$$

The compressor shaft horsepower is, of course, equal to the gas horsepower divided by the mechanical efficiency. For most centrifugal compressor applications, the mechanical losses are relatively small, and an average mechanical efficiency of 99% may be used for estimating purposes.

The rotative speed of a centrifugal compressor is fixed by the peripheral velocity of the impellers and their diameter. As indicated previously, the peripheral velocity is determined by the head to be developed; the impeller diameter is determined by the capacity to be handled, as measured at suction conditions.

From Eq. (11.10):

$$u = \sqrt{\frac{Hg}{\mu}} \qquad \text{also} \qquad u = \frac{\pi DN}{720} \qquad \text{ft/s}$$

where H = head per stage, ft
N = rotative speed, r/min
D = impeller diameter, in.
$u = \pi DN/60$ m/s, where D and H are expressed in meters

$$N = \frac{720u}{\pi D} = \frac{720\sqrt{H_g/\mu}}{\pi D} \tag{11.13}$$

$$N = \frac{1300}{D}\sqrt{\frac{H}{\mu}} \qquad \text{rpm}$$

where D and H = in. and ft, respectively, and

$$N = 59.82/D\sqrt{\frac{H}{\mu}} \qquad \text{rpm}$$

where D and H are expressed in meters. As previously indicated, for approximate calculations an average pressure coefficient of 0.55 may be assumed.

The discharge temperature for an uncooled compression process may be calculated from the fundamental relation

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^M \tag{11.14}$$

For those applications where the discharge temperatures for an uncooled compression process would be prohibitive, internal diaphragm cooling or external interstage cooling may be used. If internal diaphragm cooling is used, the average exponent of compression is approximated by the isentropic exponent, and the head may be estimated on the basis of this. With external interstage cooling, each stage, or compressor body, is dealt with separately.