
17

HEAD-FLOW CURVE SHAPE OF CENTRIFUGAL COMPRESSORS*

Much has been written concerning what goes on in a centrifugal compression stage. Unfortunately, most such literature has been produced by development people for the benefit of other development people and has been presented mostly in bits and pieces in widely scattered technical papers.

The operating supervisor at the plant level and the equipment specialist at the planning level have a real need for an overall understanding of this subject. Such knowledge can help operators to better understand the potentials and limitations of their machines. It should help specialists to determine what can realistically be expected of centrifugal compressors, and it should help in their analysis of competitive offerings.

It is on this premise that the following paragraphs are written: in the hope that in a small way at least, they can build a bridge of comprehension between the centrifugal compressor investigator and the compressor user.

17.1 COMPRESSOR STAGE

This discussion will largely concern itself with the conventional compressor stage [i.e., a radial inlet closed impeller running at 700 to 900 ft/s (213 to 273 m/s) tip speed, feeding a vaneless diffuser]. However, sufficient attention will be given to such variations as inducer impellers and vaned diffusers that a general understanding of most combinations of commonly used hardware should result.

* Developed and contributed by Donald C. Hallock, Centrifugal compressors: the shape of the curve, *Compressor Refresher*, Elliott Company, Jeannette, Pa. (Reprint 93). Originally published in *Air and Gas Engineering*, Vol. 1, No. 1, Jan. 1968, and presented through the courtesy of CAGI.

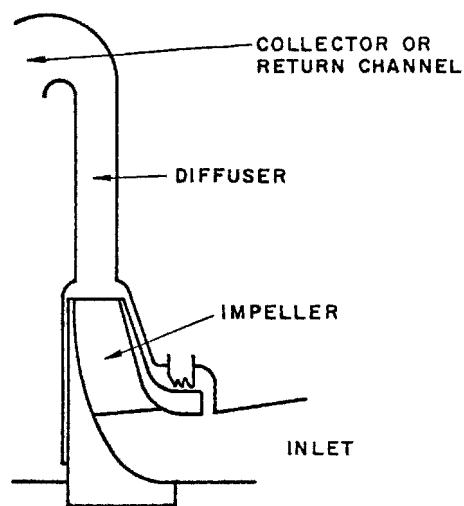


FIGURE 17.1 Impeller and diffuser geometry influence compressor performance curve. (*Elliott Company, Jeannette, Pa.*)

Discussion will center on the *impeller* and *diffuser*, because these are the two key elements in producing characteristic shape (see Fig. 17.1). Poorly designed inlets, collectors, or return channels can naturally affect performance, but their influence on characteristic shape is usually small and will henceforth be ignored.

This discussion is directly applicable to a single-stage machine and to each stage of a multistage machine. The approach taken will be largely qualitative rather than quantitative, because it is not our purpose to produce a design manual, but rather, to produce understanding. A general familiarity of the reader with centrifugal equipment is assumed. Certainly, the preceding pages of this book will be quite helpful in this regard.

17.2 ELEMENTS OF THE CHARACTERISTIC SHAPE

Any discussion of characteristic shape must, like ancient Gaul, be divided into three parts. We have a *basic slope* of head vs. flow, upon which we must superimpose a *choke* or *stonewall* effect in the overload region and a minimum flow or *surge* point in the underload region. The resulting overall characteristic will then be the basic slope as altered and limited by choke at high flow and as limited by surge at low flow in Fig. 17.2. We discuss each of the three parts in turn.

17.2.1 Basic Slope

To understand *basic slope*, it is necessary to look at what is going on at the impeller tip in terms of velocity vectors. In Fig. 17.3, V_{rel} represents the gas velocity relative to the blade. U_2 represents the absolute tip speed of the blade. The resultant of these two vectors is represented by V , which is the actual absolute velocity of the gas. (By vector addition, $U_2 + V_{\text{rel}} = V$.) It can be seen that the length of the vectors and the magnitude of the exit angle α are determined by the amount of backward lean in the blade, by the tip speed of the blade, and by gas velocity relative to the blade, which is in turn dictated by tip-volume-flow rate for a given impeller.

Having the magnitude and direction of the absolute velocity V , we now break this vector into its radial and tangential components, V_r and V_t , as in Fig. 17.4. The vector V_t is reduced somewhat by the slip factor in a real impeller, an effect that can be ignored in a qualitative

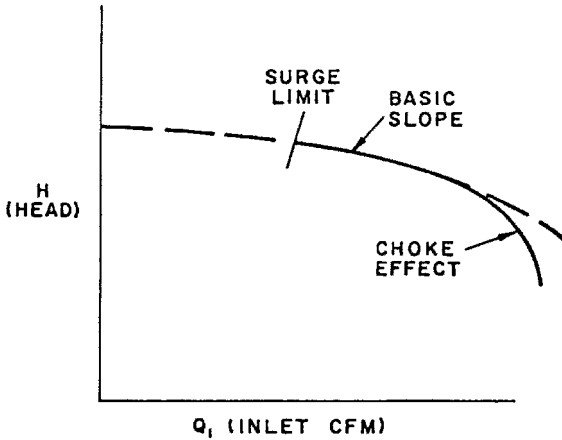


FIGURE 17.2 Characteristic shape of a centrifugal compressor performance curve. (Elliott Company, Jeannette, Pa.)

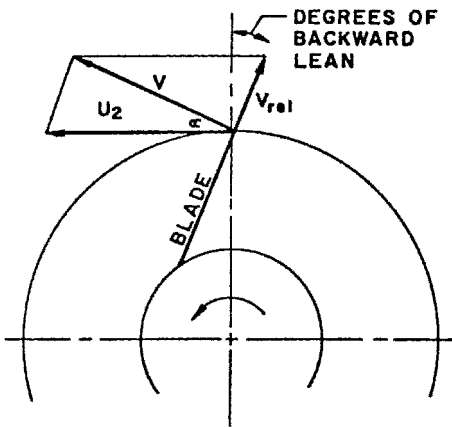


FIGURE 17.3 Blade angle and velocity relationships. (Elliott Company, Jeannette, Pa.)

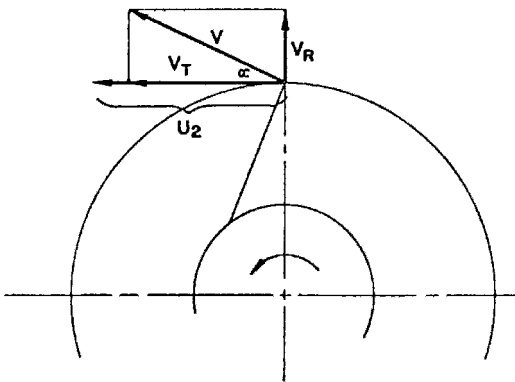


FIGURE 17.4 Resolution of vectors. Note that head output is proportional to product $V_T U_2$. (Elliott Company, Jeannette, Pa.)

discussion such as this. The head output is proportional to the product of $U_2 V_T$. For a given rpm rate, U_2 is constant; therefore, head is proportional to V_T .

Let us now look at what happens to the magnitude of the tangential component V_T as we vary the amount of flow passing through the impeller at constant rpm. As the flow is decreased,

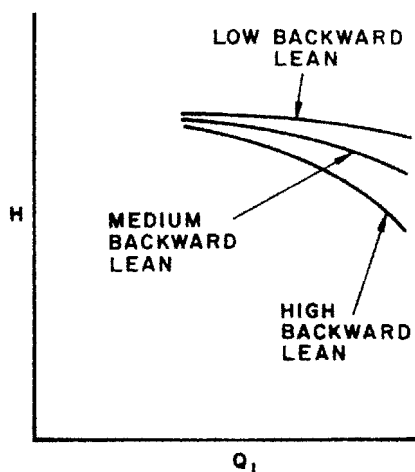


FIGURE 17.5 Effect of blade inclination on head rise. (Elliott Company, Jeannette, Pa.)

V_{rel} decreases. As V_{rel} decreases, angle α decreases markedly. This makes V_t increase, which increases head output. This head increase with decreasing flow is the *basic slope* of the stage characteristic.

17.2.2 Blade Angle

How does the degree of backward lean affect the steepness of the basic slope? Picture a radial blade (zero backward lean). V_{rel} is now the same as V_r in Fig. 17.4 and V_t is now equal to U_2 . As we reduce the flow in this impeller, V_r and α decrease as before. V_t remains constant, however. Head output therefore remains theoretically constant, regardless of flow. In a real impeller, of course, the head is reduced on increasing flow by a decrease in efficiency attributable to higher frictional losses. The resulting basic slope normally shows a 2% or 3% head rise when going from design flow to minimum flow.

Now let us look at the opposite extreme, an impeller having a very high degree of backward lean: say, 45° off radial at the tip. We can see that a change in flow, and therefore a change in the V_{rel} vector length, will cause very large changes in V_t , and therefore in head. Thus, such an impeller will typically produce a head rise of 20% or more when moving from design flow to minimum flow.

It is evident from the foregoing that the effect of backward lean on head output is minimized at low flow; a high-backward-lean impeller will produce almost as much head at minimum flow as will a low-backward-lean impeller running at the same tip speed. As we move out toward design flow, however, the head difference becomes quite dramatic, as shown in Fig. 17.5. The normal industry standard for conventional closed impellers is represented by the middle line, which is 25 to 35° of backward lean. This configuration is really a compromise between the high head obtainable at design flow with low-backward-lean blades and the steep basic slope obtainable with high-backward-lean blades.

One further point should be made concerning basic slope before we leave the subject. In the foregoing discussion, we used the term *flow* without elaboration, the implication being that impeller-tip-volume rate is dictated by inlet volume rate regardless of the rotative speed and type of gas. This, of course, is not quite true—gases, unlike liquids, being compressible.

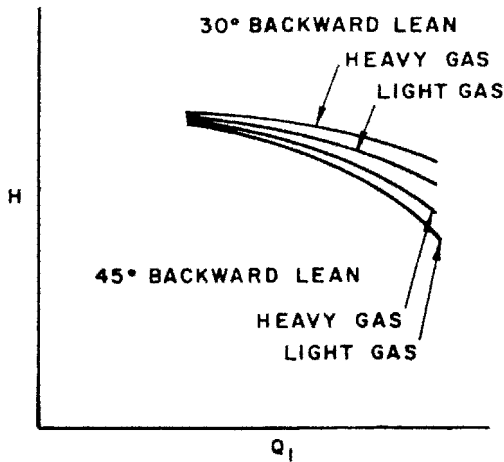


FIGURE 17.6 Effect of molecular weight on the slope of a performance curve. (*Elliott Company, Jeannette, Pa.*)

It is well known that a heavy gas will be compressed to a greater extent in a given stage than a light gas (i.e., the heavy gas has a higher volume ratio).

Therefore, for a given *inlet* cfm entering a given impeller at a given speed, the magnitude of V_{rel} is less for a heavy gas than for a light gas. If the impeller has backward lean, the magnitude of V_t will be greater for the heavy gas. Since head output is proportional to V_t , a given impeller running at a given speed will produce more head when compressing a heavy gas than when compressing a like *inlet* cfm of light gas. What is more, the magnitude of the difference increases at inlet flow increases, so the basic slope of a given backward lean impeller is actually less steep for a heavy gas than for a light gas. The higher the backward lean, the more pronounced this effect (see Fig. 17.6).

17.2.3 Fan Law Effect

The effect of volume ratio on what is known as *fan law* is worthy of mention. The fan law states that the cfm potential of a stage is proportional to the rotative speed and that the head produced is proportional to speed squared. Reexamination of Figs. 17.3 and 17.4 will demonstrate the logic of this law.

If V_{rel} were truly proportional to *inlet* cfm and we increased both inlet cfm and speed by 10%, the head output would be 21% greater, because the tip-vector geometry would maintain exact similarity. Higher head produces a higher volume ratio in a given gas; however, V_{rel} does not increase quite in proportion to speed and inlet cfm. By reasoning similar to that used in discussing heavy gas vs. light gas, then, the head output of a backward-leaning stage handling 10% more inlet cfm at 10% higher speed will increase somewhat *more* than 21%. By similar reasoning, if we reduce speed and inlet flow from 100% to 90%, the head produced will be slightly *less* than the 81% predicted by the fan laws, (Table 17.1).

The *fan laws* (affinity laws) acquired their name from the fact that a fan is a low-head compressor normally handling air, a light gas. Since volume ratio effects are extremely small when imparting a small head to light gas, excellent accuracy can be obtained by the fan laws. As a general rule, the higher the head, the heavier the gas, and the greater the backward lean, the poorer the accuracy obtained by the fan laws will be. As a practical matter, speed changes up to 30 or 40% can be handled with sufficient accuracy for most purposes when dealing with typical single-stage air compressors. A little more discretion must be used on multistage

TABLE 17.1 Fan Laws (Affinity Laws)

1. With impeller diameter D held constant:

A. $\frac{Q_1}{Q_2} = \frac{N_1}{N_2}$

B. $\frac{H_1}{H_2} = \left(\frac{N_1}{N_2} \right)^2$

C. $\frac{\text{bhp}_1}{\text{bhp}_2} = \left(\frac{N_1}{N_2} \right)^3$

where Q = capacity, cfm

H = total head, ft

bhp = brake horsepower

N = compressor speed, rpm

2. With speed N held constant:

A. $\frac{Q_1}{Q_2} = \frac{D_1}{D_2}$

B. $\frac{H_1}{H_2} = \left(\frac{D_1}{D_2} \right)^2$

C. $\frac{\text{bhp}_1}{\text{bhp}_2} = \left(\frac{D_1}{D_2} \right)^3$

When the performance (Q_1 , H_1 , and bhp_1) is known at some particular speed (N_1) or diameter (D_1), the formulas can be used to estimate the performance (Q_2 , H_2 , and bhp_2) at some other speed (N_2) or diameter (D_2). The efficiency remains nearly constant for speed changes and for small changes in impeller diameter.

compressors handling heavy gases, however, because fan law deviation can become quite significant for a speed change as small as 10%.

17.2.4 Choke Effect

We have discussed at some length the basic slope of the head-flow curve and have avoided until now the choke or stonewall effect that occurs at flows higher than design and that must be superimposed on the basic slope, as in Fig. 17.2.

Just as basic slope is controlled by impeller-*tip*-vector geometry, the stonewall effect is normally controlled by impeller-*inlet*-vector geometry. In Fig. 17.7 we can draw vector U_1 to represent the tangential velocity of the leading edge of the blade (similar to U_2 at the tip). We can also draw vector V , representing absolute velocity of the inlet gas, which having made a 90° turn is now moving essentially radially—hence the term *radial inlet*. By vector analysis, V_{rel} , which is gas velocity relative to the blade, is of the magnitude and direction

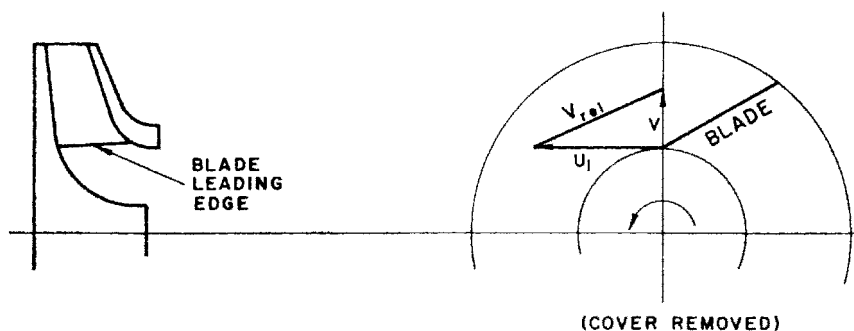


FIGURE 17.7 Velocity relationships at the leading edge of an impeller blade. (Elliott Company, Jeannette, Pa.)

shown, where $U_1 + V_{\text{rel}} = V$. At design flow, the direction of V_{rel} essentially lines up with the blade angle as shown.

17.2.5 Mach Number

The magnitude of V_{rel} compared to the speed of sound at the inlet is called the *relative inlet Mach number*. It is the magnitude of this ratio that dictates stonewall in a conventional stage. Although true stonewall should theoretically not be reached until the relative inlet Mach number is unity, it is conventional practice not to exceed 0.85 or 0.90 at design flow.

It is evident from Fig. 17.7 that for a given rpm, the magnitude of V_{rel} will diminish with decreasing flow, since V is proportional to flow. If V_{rel} decreases, relative inlet Mach number decreases, so the stonewall effect is normally not a factor at flows below design. It is also evident that at low flows, the direction of V_{rel} is such that the gas impinges on the leading side of the blade (positive incidence), a factor that is not very detrimental to performance until very high values of positive incidence are reached.

Let us now *increase* flow beyond the design point. As V increases, so do V_{rel} and relative inlet Mach number. In addition, V_{rel} now impinges on the *trailing* side of the blade, a condition known as *negative incidence*. It has been observed that high degrees of negative incidence tend to contribute to the stonewall problem as Mach 1 is approached, presumably because of boundary layer separation and reduction of effective flow area in the blade pack.

17.2.6 Significance of Gas Weight

Since values of U_1 are typically in the range 500 ft/s (152 m/s) and values of V in the range 250 ft/s (76 m/s), it is obvious that air at the speed of sound at 80°F (27°C) = 1140 ft/s (347 m/s), and lighter gases suffer no true impeller stonewall problems as described earlier, even at high overloads. Some head loss below the basic slope will be observed, however, in even the lightest gases, in part because of increased frictional losses throughout the entire stage and the extreme negative incidence at high overloads.

The lightest common gas handled by conventional centrifugals for which stonewall effect can be a definite factor is propylene with a speed of sound of 740 ft/s (225 m/s) at -40°F

(-40°C). In order of increasing severity are propane at 718 ft/s (219 m/s) at -40°F (-40°C), butane, chlorine at 630 ft/s (192 m/s) at -20°F (-29°C), and the various Freons. The traditional method of handling such gases is to use an impeller of larger than normal flow area—to reduce V —and run it at a lower than normal rpm value—to reduce U_1 —thus keeping the value of V_{rel} abnormally low. This procedure requires the use of more than the usual number of stages for a given head requirement and sometimes even requires the use of an abnormally large frame size for the flow handled.

17.2.7 Inducer Impeller Effects on Head Output

Much development work has been done in recent years toward the goal of running impellers at normal speeds on heavy gases to reduce hardware costs to those incurred in the compression of light gases. One approach has been to use inducer impellers, as in Fig. 17.8. The blades on this impeller extend down around the hub radius so that the gas first encounters the blade pack while flowing axially. Figure 17.8 shows the vector analysis at the inducer's outer radius. Assuming that the inducer radius is the same as the leading-edge radius of a conventional radial inlet impeller, the vector geometries of the two are identical.

The advantage of the inducer lies in the fact that as we move radially inward along the blade leading edge, the value of U_1 (and, therefore, of V_{rel} and Mach number) decreases. As we move along the leading edge of a *conventional* impeller, the vector geometry remains essentially constant. It can be seen, therefore, that whereas the *maximum* Mach number for the two styles is the same, the *average* Mach number for the inducer is *less* for a given flow and speed. The inducer impeller can therefore be run somewhat faster, resulting in greater head output. The big disadvantage of a closed inducer impeller lies in the difficulty of fabrication. It is obviously more difficult to weld the longer and more curved blade path of an inducer impeller than that of a conventional impeller. Other disadvantages are the greater weight and greater axial space requirement of an inducer impeller over that of a conventional impeller.

Another method of obtaining increased head output for a given Mach number is the reduction of backward lean. This expedient has some disadvantages, however, not the least of which is the flatter curve that results.

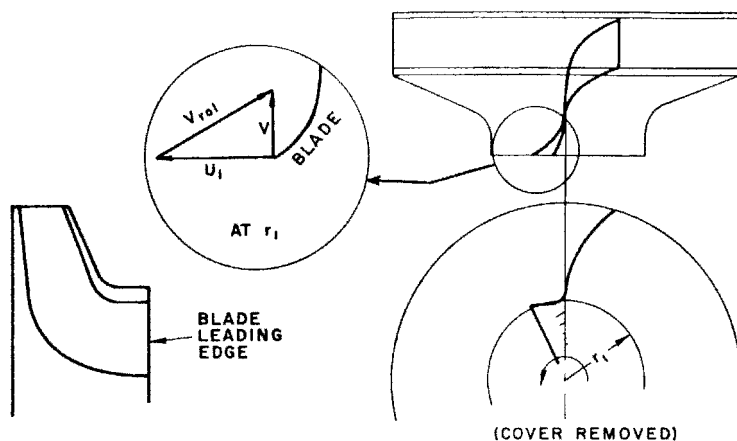


FIGURE 17.8 Vector diagrams for inducer impellers. (Elliott Company, Jeannette, Pa.)

17.2.8 Surge

Earlier in the book, surge was discussed from the point of view of compressor control. But having discussed *basic slope* and *choke*, we are left with a vector-based discussion of minimum flow, or *surge*.

Surge flow has been defined by some as the flow at which the head-flow curve is perfectly flat and below which head actually decreases. This definition has a certain appeal, because straddling a surge flow so defined are myriad pairs of flow values producing identical heads, leading one to conjecture that the flow value is actually jumping back and forth between such a pair. However, since numerous centrifugal stages have been observed to run smoothly at flows below such a rate and others to surge at flows above such a rate, this definition must be considered imperfect at best. We must recall that unlike choke flow, which hurts nothing but aerodynamic performance, surge can be quite damaging to a compressor and should be avoided. The higher the pressure level involved, the more important this statement becomes.

To understand what causes surge in a conventional stage, we must refer back to the tip vector geometry of Fig. 17.4. Because flow is reduced while speed is held constant, the magnitude of V_r decreases in proportion, and that of V_t remains constant for radial blades or increases for backward-lean blades. As flow decreases, therefore, the value of flow angle α decreases. In the normal parallel wall vaneless diffuser, this angle remains almost constant throughout the diffuser, so the path taken by a “particle” of gas is a log spiral in Fig. 17.9. The reason that angle α remains constant in a parallel wall diffuser is that both V_r and V_t vary inversely with radius— V_r because radial flow area is proportional to radius and V_t because of the law of conservation of momentum.

It is evident from Fig. 17.9 that the smaller the angle α , the longer the flow path of a given gas particle between the impeller tip and the diffuser outer diameter. When angle α becomes small enough and the diffuser flow path long enough, the flow momentum at the walls is dissipated by friction to the point where pressure gained by diffusion causes a reversal of flow, and *surge results*. The angle α at which this occurs in a vaneless diffuser has been found to be quite predictable for various diffuser–impeller diameter ratios. The flow (and angle α) at which surge occurs can be lowered somewhat by reducing diffuser diameter but at the cost of some velocity pressure recovery.

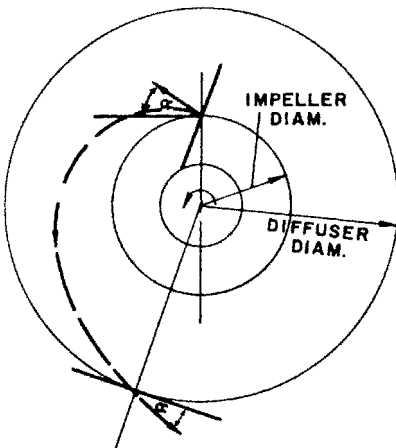


FIGURE 17.9 Spiral path taken by a molecule of gas in an impeller. (Elliott Company, Jeannette, Pa.)

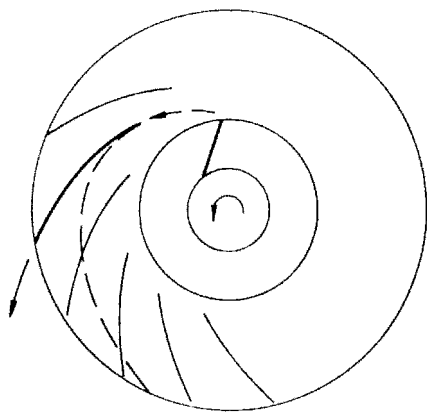


FIGURE 17.10 Vaned diffuser surrounding backward-leaning impeller vanes. (Elliott Company, Jeannette, Pa.)

17.2.9 Vaned Diffusers

Before we discuss the foregoing in more detail, let us briefly discuss *vaned diffusers*, devices sometimes used in high-performance machines. Figure 17.10 shows the configuration of diffuser vanes. The vanes force the gas outward in a shorter path than unguided gas would take but not so short a path as to cause too rapid deceleration, with consequent stream separation and inefficiency.

The leading edge of the diffuser vane is set for shockless entry of the gas at approximately design flow. It is evident that at flows lower than design, the gas impinges on the diffuser vanes with positive incidence. Conversely, at flows higher than design, negative incidence prevails. In a typical high-speed high-performance stage, positive incidence at the leading edge of the diffuser vane triggers surge on decreasing flow. On increasing flow, negative incidence at the inducer vanes can cause choking before impeller-inlet stonewall is reached. Despite this disadvantage, vaned diffusion is sometimes used for air and certain other gases because stage efficiency is improved by 2 to 3%. The short-flow range problem can, of course, be alleviated by making the diffuser vanes adjustable.

17.2.10 Vaneless Diffusers

Having made our obeisance to vaned diffusion, let us return to the more common vaneless diffuser. We have seen that when V_r and α become too small, we will have surge. What can we do if our parameters are such that we are faced with a low value for α at design flow? We can increase V_r and α artificially by pulling our diffuser walls together until α reaches the proper value at design flow. This brings to light an important distinction: head output, as discussed earlier, is controlled by vector geometry in the *impeller tip* largely irrespective of what happens in the diffuser. Surge point is controlled by vector geometry in the *diffuser*, largely irrespective of what occurred in the impeller. In the common case where impeller tip width and diffuser width are the same (Fig. 17.1), the two sets of vector geometry are the same—ignoring impeller blade solidity. If such a stage has poor stability, it is frequently possible to lower the surge point by narrowing the diffuser without markedly changing basic slope or choke flow. This procedure can be carried only so far, however, because extreme positive incidence at the impeller inlet will eventually trigger surge regardless of diffuser geometry.

Just as we did when discussing choke flow, let us look at the effect of heavy gas compression on surge point. Since a heavy gas is compressed more at a given speed than is a

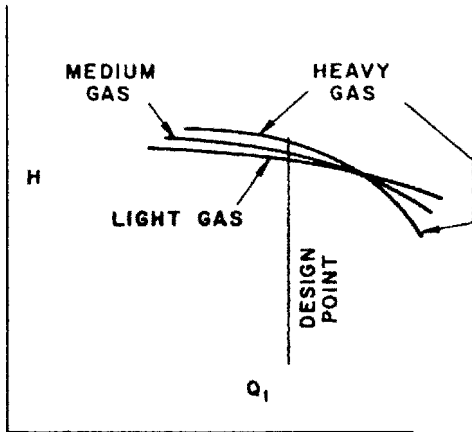


FIGURE 17.11 Head-flow characteristics of a given stage operating on different molecular weight gases. (Elliott Company, Jeannette, Pa.)

light gas, it is evident that the critical value for α will be reached (on decreasing flow) at a higher *inlet* flow of heavy gas than that of a light gas. A given stage therefore has a higher surge flow or a lower stable range when compressing heavy gas than when compressing light gas at the same speed.

By similar reasoning, a given stage compressing a given gas at varying speed will surge at somewhat different inlet flows than those predicted by fan law. When speed is 10% above design speed, for instance, surge flow will be *more* than 10% higher than surge flow at design speed. When speed is 90% of design, the stage will surge at *less* than 90% of design-speed surge flow.

We have discussed in some detail the three ingredients involved in a centrifugal compressor characteristic: basic slope, choke, and surge. We have considered how various physical design parameters such as backward lean, inlet blade angle, and diffuser flow angle affect these ingredients. We have also discussed the effect on characteristic shape of compressing different weight cases in a given stage. Now let us consolidate these bits and pieces into an overall look at the characteristic of a given stage used on various gases at various speeds.

In Fig. 17.11 we plotted the head-flow characteristic for a given conventional stage running at a given speed on various gases. There should be no surprises here because we have discussed all the effects shown. Now if we divide the ordinate H by the speed squared, and the abscissa Q by speed, we have the same qualitative set of characteristics, except that heavy gas becomes high speed and light gas becomes low speed in Fig. 17.12. This figure illustrates departure from the fan law.

17.2.11 Equivalent Tip Speeds

It is possible and quite appropriate to express Figs. 17.11 and 17.12 as a single plot. To do so, it is only necessary to use nomenclature that includes both speed and type of gas. A convenient method of doing this is to multiply tip speed by the ratio of some reference acoustic velocity to actual gas acoustic velocity and call the resulting number *equivalent tip speed* (Fig. 17.13).

The reference acoustic velocity normally used is air at 80°F (27°C), since this is the gas on which components are usually tested. Using the equivalent tip speed concept permits the

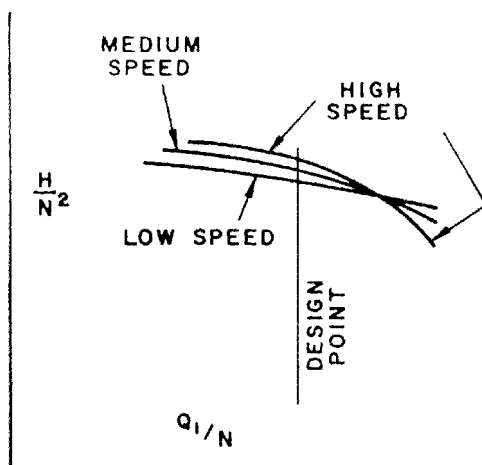


FIGURE 17.12 Speed-based performance curves illustrating departure from the fan laws. (Elliott Company, Jeannette, Pa.)

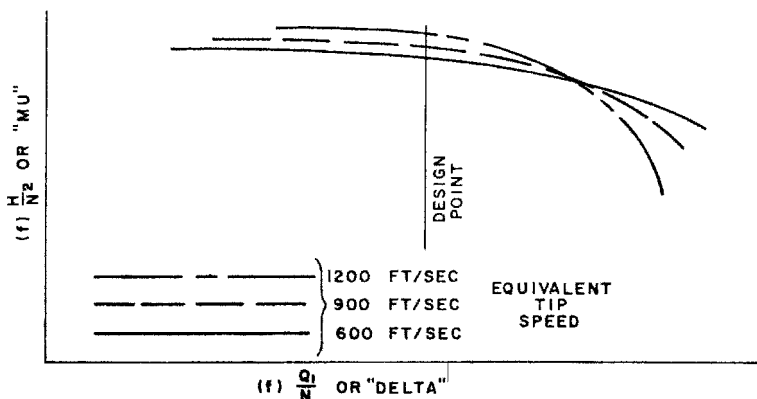


FIGURE 17.13 Plots of head coefficient (" μ ") vs. flow coefficient (" δ ") allow prediction of stage performance with different gases. (Elliott Company, Jeannette, Pa.)

use of a single set of curves to describe the characteristic of a *given* stage when compressing any gas at any speed.

By way of example, we mentioned earlier that the sonic velocity of air at 80°F is 1140 ft/s and that of propylene at -40°F is 740 ft/s. If a stage is running at a tip speed of 780 ft/s on propylene, the *equivalent* tip speed is $780 \times 1140/740$ or 1200 ft/s (366 m/s). The stage characteristic *shape* obtained at 780 ft/s tip speed on propylene is therefore the same as that obtained at 80°F air at 1200 ft/s (366 m/s) tip speed, but the head output of the latter is of course much higher.

The ordinate H/N^2 is commonly adjusted by some constants and called *head coefficient*, or *mu* (μ). The abscissa Q/N is likewise adjusted by constants and called *flow coefficient*, or *delta* (δ). Figure 17.13 is in actuality, then, the common μ - δ curve used by some compressor investigators to predict stage performance. Others call the head coefficient " ϕ " (ϕ), and the flow coefficient " ψ " (ψ); see Sections 11.5 and 12.9.

17.3 CONCLUSIONS

Let us now review in practical terms just what we have learned. We now know that if conventional machinery is run at the usual tip speeds of 800 to 900 ft/s (244 m/s to 274 m/s) on propylene or heavier gases, the characteristic shape will be quite flat between design and surge, the stable range will be low, and the overload capacity almost nil. We can also see that even between 700 and 750 ft/s (213 m/s and 229 m/s) tip speed—the usual range selected for propylene, propane, and butane—we simply cannot expect the traditional 40 to 50% stable range and generous overload capacity obtainable on air machines. To obtain such a characteristic, it would be necessary to run between 500 and 600 ft/s (152 and 183 m/s), which would almost double the number of stages required for a given amount of compression! The 700 to 750 ft/s range normally used is obviously a compromise between practical economics and desirable characteristic shape and range.

It is hoped that the foregoing has helped in a small degree to close the gap between the specialist and the generalist without too greatly offending the former or too greatly confusing the latter. Compressor performance is a difficult subject at best, and even today new insights are being gained through continuing development programs.