UNIT 6 KEYS AND COUPLINGS

Structure

6.1 INTRODUCTION

A key is a piece of steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Objectives

After studying this unit, you should be able to

- identify keys and their application,
- calculate forces on keys, and
- design keys.

6.2 TYPES OF KEYS

The following types of keys are important from the subject point of view :

(a) Shunk keys,

(b) Saddle keys,

(c) Tangent keys,

(d) Round keys, and

(e) Splines.

We shall now discuss the above types of keys, in detail, in the following sections.

6.2.1 Sunk Keys

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley or gear. The sunk keys are of the following types :

Rectangular Sunk Key

A rectangular sunk key is shown in Figure 6.1. The usual proportions of this key are :

Width of key, $w = \frac{d}{4}$; and thickness of key, $t = \frac{2w}{3} = \frac{d}{6}$

where d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only.

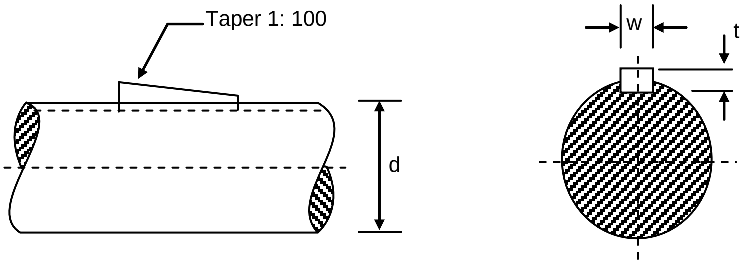


Figure 6.1 : Rectangular Sunk Key

Square Sunk Key

The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e.

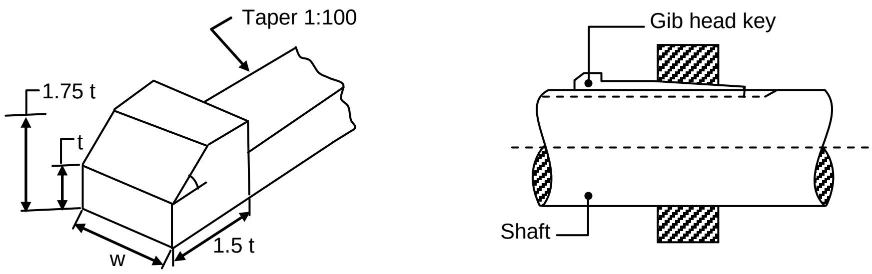
$$w = t = \frac{d}{4}$$

Parallel Sunk Key

The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating part is required to slide along the shaft.

Gib-head Key

It is a rectangular sunk key with a head at one end known as **gib head**. It is usually provided to facilitate the removal of key. A gib head key is shown in Figure 6.2(a) and its use in shown in Figure 6.2(b).



(a) **(b)**

Figure 6.2 : Gib-head Key

The usual proportions of the gib head key are :

Width, $w = \frac{d}{4}$;

and thickness at large end, $t = \frac{2w}{3} = \frac{d}{6}$.

Feather Key

A key attached to one member of a pair and which permits relative axial movement of the other is known as **feather key**. It is a special key of parallel type which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.

The feather key may be screwed to the shaft as shown in Figure 6.3(a) or it may have double gib heads as shown in Figure 6.3(b). The various proportions of a feather key are same as those of rectangular sunk key and gib head key.

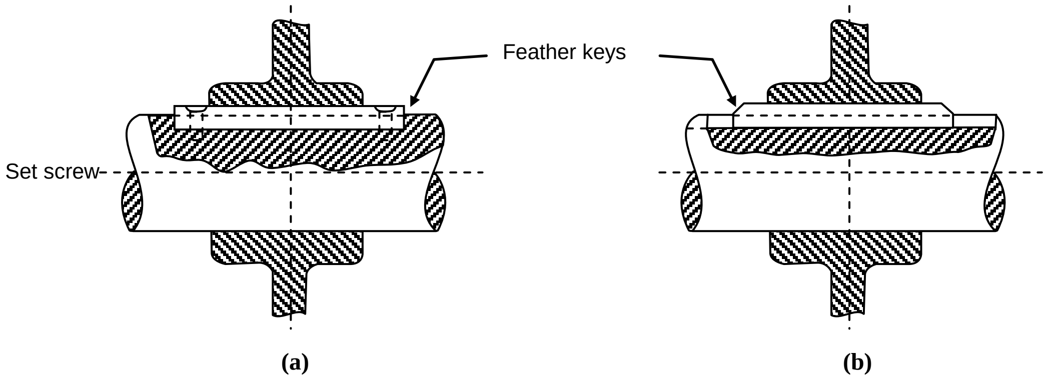


Figure 6.3 : Feather Key

The following Table 6.1 shows the proportions of standard parallel, tapered and gib head keys, according to IS : 2292 and 2293-1974 (Reaffirmed 1992).

Table 6.1 : Proportions of Standard Parallel, Tapered and Gib Head Key

Shaft Diameter (mm) upto and Including	Key Cross-section		Shaft Diameter (mm) upto and Including	Key Cross-section	
	Width (mm)	Thickness (mm)		Width (mm)	Thickness (mm)
6	2	2	85	25	14
8	3	3	95	28	16
10	4	4	110	32	18
12	5	5	130	36	20
17	6	6	150	40	22
22	8	7	170	45	25
30	10	8	200	50	28
38	12	8	230	56	32
44	14	9	260	63	32
50	16	10	290	70	36
58	18	11	330	80	40
65	20	12	380	90	45
75	22	14	440	100	50

Woodruff Key

The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Figure 6.4. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

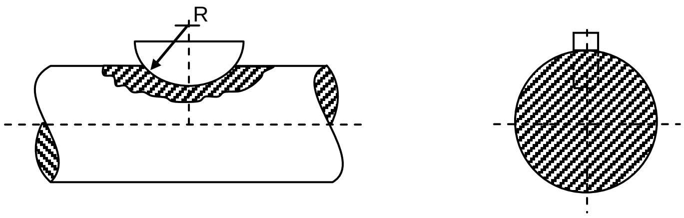


Figure 6.4 : Woodruff Key

The main advantages of a woodruff key are as follows :

(a) It accommodates itself to any taper in the hub or boss of the mating piece.

(b) It is useful on tapering shaft ends. Its extra depth in the shaft prevents any tendency to turn over in its keyway.

The disadvantages are :

(a) The depth of the keyway weakens the shaft.

(b) It can not be used as a feather.

6.2.2 Saddle Keys

The saddle keys are of the following two types :

(a) Flat saddle key, and

(b) Hollow saddle key.

A **flat saddle key** is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in Figure 6.5. It is likely to slip round the shaft under load. Therefore, it is used for comparatively light loads.

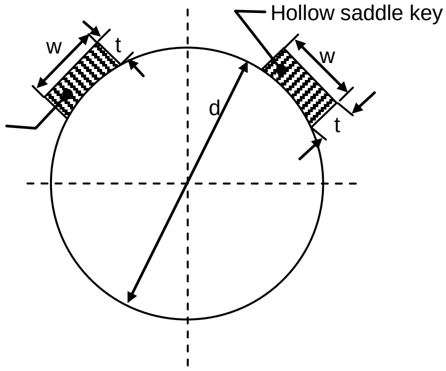


Figure 6.5 : Saddle Key

A **hollow saddle key** is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore, these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams, etc.

6.2.3 Tangent Keys

The tangent keys are fitted in pair at right angles as shown in Figure 6.6. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

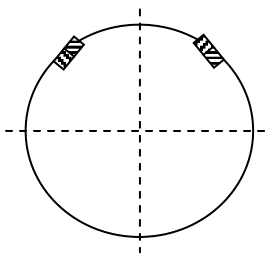
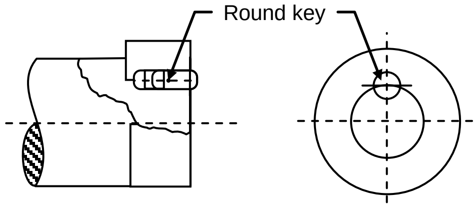


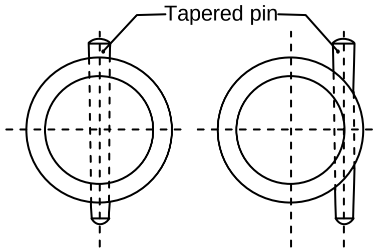
Figure 6.6 : Tangent Keys

6.2.4 Round Keys

The round keys, as shown in Figure 6.7(a), are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage of manufacturing as their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.



(a)



(b)

Figure 6.7 : Round Keys

Sometimes the tapered pin, as shown in Figure 6.7(b), is held in place by the friction between the pin and the reamed tapered holes.

6.2.5 Splines

Sometimes, keys are made integral with the shaft which fit in the keyways broached in the hub. Such shafts are known as **splined shafts** as shown in Figure 6.8. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.

The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive.

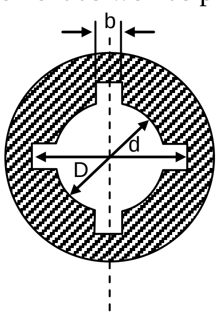


Figure 6.8 : Splines

6.3 FORCE ACTING ON A SUNK KEY

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

(a) Forces (F1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

(b) Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub.

The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Figure 6.9.

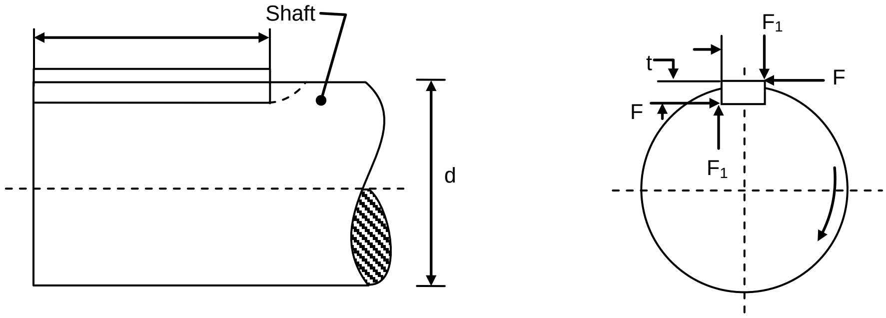


Figure 6.9 : Forces Acting on a Shunk Key

In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

6.4 STRENGTH OF A SUNK KEY

A key connecting the shaft and hub is shown in Figure 6.9.

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key,

t = Thickness of key, and

τ and σ_c = Shear and crushing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

$$F = \text{Area resisting shearing} \times \text{Shearing stress} = l \times w \times \tau$$

$\therefore$ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \qquad \ldots (6.1)$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

$\therefore$ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \ldots (6.2)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \text{[Equating Eqs. (6.1) and (6.2)]}$$

or $$\frac{w}{t} = \frac{\sigma_c}{2\tau} \qquad \text{. . . (6.3)}$$

The permissible crushing stress for the usual key material is atleast twice the permissible shearing stress. Therefore, from Eq. (6.3), we have $w = t$. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

The torque transmitted by the key,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \text{. . . (6.4)}$$

and the torque transmitted by the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \qquad \text{. . . (6.5)}$$

(Taking τ_1 = Shear stress for the shaft material)

From Eqs. (6.4) and (6.5), we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571 d \times \frac{\tau_1}{\tau} \qquad \left(\text{Taking } w = \frac{d}{4}\right) \qquad \text{. . . (6.6)}$$

When the key material is same as that of the shaft, then $\tau = \tau_1$.

$\therefore$ $l = 1.571\, d$ [From Eq. (6.6)]

Example 6.1

Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MP and 70 MPa.

Solution

Given d = 50 mm; τ = 42 MPa = 42 N/mm^2; σ = 70 MPa = 70 N/mm^2.

The rectangular key is designed for a shaft of 50 mm diameter,

Width of key, $w = \frac{d}{4}$, w = 12.5 mm

and thickness of key as $\frac{d}{6}$ t = 8.3 mm

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 17 \times 42 \times \frac{50}{2} = 13125\, l \text{ N-mm} \qquad \text{. . . (6.7)}$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\ (50)^3 = 1.03 \times 10^6 \text{ N-mm} \qquad \text{. . . (6.8)}$$

From Eqs. (6.7) and (6.8), we have

$$l = 1.03 \times \frac{10^6}{13125} = 79.25 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{8.3}{2} \times 70 \times \frac{50}{2} = 7262.5\, l \text{ N-mm} \quad \ldots (6.9)$$

From Eqs. (6.8) and (6.9), we have

$$l = 1.03 \times \frac{10^6}{8750} = 141.8 \text{ mm}$$

Taking larger of the two values, we have length of key,

l = 141.8 say 142 mm.

Example 6.2

A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm width and 9 mm thickness made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.

Solution

Given d = 45 mm; σ_γ for shaft = 400 MPa = 400 N/mm^2; w = 14 mm; t = 9 mm; σ_{yt} for key = 340 MPa = 340 N/mm^2.

Let l = Length of key.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{\sigma_\gamma}{2 \times F.S.} = \frac{400}{2 \times 2} = 100 \text{ N/mm}^2$$

and maximum shear stress for the key,

$$\tau_k = \frac{\sigma_\gamma}{2 \times F.S.} = \frac{340}{2 \times 2} = 85 \text{ N/mm}^2$$

(**Note :** Yield strength for shaft and key materials are different).

We know the maximum torque transmitted by the shaft and key,

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3 = \frac{\pi}{16} \times 100\,(45)^3 = 1.8 \times 10^6 \text{ N-mm}$$

First of all, let us consider the failure of key due to shearing. We know that the maximum torque transmitted (T),

$$1.8 \times 10^6 = l \times w \times \tau_k \times \frac{d}{2} = l \times 14 \times 85 \times \frac{45}{2} = 26775\, l$$

$$\therefore \quad l = \frac{1.8 \times 10^6}{26775} = 67.2 \text{ mm}$$

Now considering the failure of key due to crushing. We know that the maximum torque transmitted by the shaft and key (T),

$$1.8 \times 10^6 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} = 17213\, l \quad \left(\text{Taking } \sigma_{ck} = \frac{\sigma_\gamma}{F.S.}\right)$$

$$\therefore \quad l = \frac{1.8 \times 10^6}{17213} = 104.6 \text{ mm}$$

Taking the larger of the two value, we have

l = 104.6 say 105 mm.

6.5 EFFECT OF KEYWAYS

A little consideration will show that the keyway cut in the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. In other words, the torsional strength of the shaft is reduced. The following relation for the weakening effect of the keyway is based on the experimental results by H. F. Moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

where k_e = Shaft strength reduction factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway.

w = Width of keyway,

d = Diameter of shaft, and

h = Depth of keyway $= \dfrac{\text{Thickness of key } (t)}{2}$.

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

In case the keyway is too long and the key is of sliding type, then the angle of twist is increased in the ratio k_θ as given by the following relation :

$$k_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)$$

where k_θ = Reduction factor for angular twist.

Example 5.3

A 15 kW, 960 rpm motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear and crushing stresses for the mild steel key are 56 MPa and 112 MPa respectively. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.

Solution

Given $P = 15$ kW $= 15 \times 10^3$ W; N = 960 rpm; $d = 40$ mm; $l = 75$ mm; $\tau = 56$ MPa $= 56$ N/mm^2; $\sigma_c = 112$ MPa $= 112$ N/mm^2.

We know that the torque transmitted by the motor,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 960} = 149 \text{ N-m} = 149 \times 10^3 \text{ N-mm}$$

Let w = Width of keyway or key.

Considering the key in shearing. We know that the torque transmitted (T).

Assuming that length of the key is equal to length of the shaft (i.e. extension)

$$149 \times 10^3 = l \times w \times \tau \times \frac{d}{2} = 75 \times w \times 56 \times \frac{40}{2} = 84 \times 10^3\, w$$

$$\therefore \quad w = \frac{149 \times 10^3}{84 \times 10^3} = 1.8 \text{ mm}$$

This width of keyway is too small. The width of keyway should be at least $\dfrac{d}{4}$.

$$\therefore \quad w = \frac{d}{4} = \frac{40}{4} = 10 \text{ mm}$$

Since $\sigma_c = 2\tau$, therefore, a square key of w = 10 mm and t = 10 mm is adopted.

According to H. F. Moore, the shaft strength factor,

$$k_e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{t}{2d}\right) \quad \text{(because } h = \frac{t}{2}\text{)}$$

$$= 1 - 0.2\left(\frac{10}{20}\right) - \left(\frac{10}{2 \times 40}\right) = 0.8125$$

$\therefore$ Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 = 75 \times 10 \times 56 \times (40)^3\ 0.8125 = 571\,844 \text{ N}$$

and shear strength of the key, i.e. torque carrying capacity

$$= l \times w \times \tau \times \frac{d}{2} = 75 \times 10 \times 56 \times \frac{40}{2} = 840\,000 \text{ N}$$

$$\therefore \quad \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = \frac{840\,000}{571\,844} = 1.47$$

6.6 COUPLINGS

In engineering applications there arise several cases where two shafts have to be connected so that power from driving shaft is transmitted to driven shaft without any change of speed. Such shafts are normally coaxial with slight or no misalignment and can be connected through devices known as couplings. Permanent couplings, often referred to as couplings, are the connectors of coaxial shafts and cannot be disengaged when shafts are running. On the other hand, those couplings which can be readily engaged or disengaged when driving shaft is running are termed as clutches. The power is transmitted when a clutch is engaged and not transmitted when clutch is disengaged. In this unit only permanent couplings will be considered. Figure 6.10 shows one such coupling connecting the shaft of an electric motor with the shaft of a worm and worm wheel reducer.

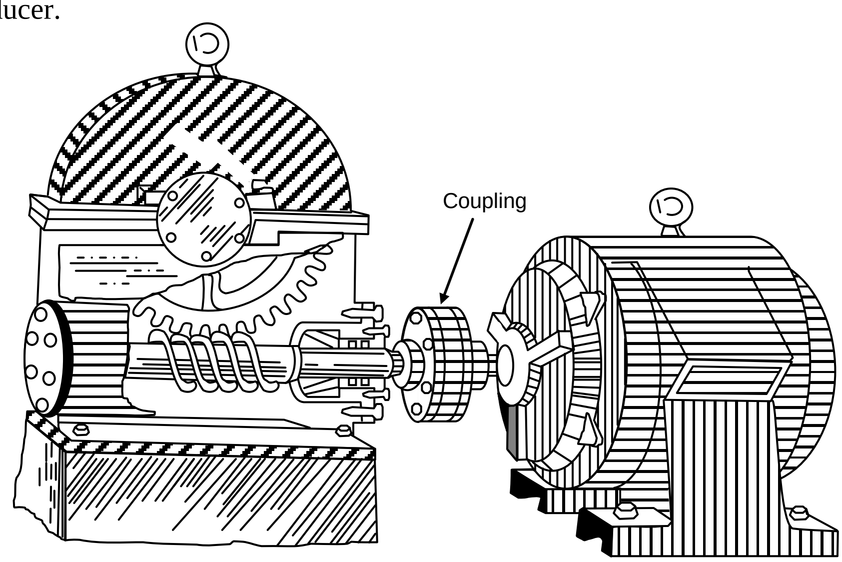


Figure 6.10 : A Permanent Coupling Connecting Coaxial Shafts of an Electric Motor and a Worm and Worm Wheel Reducer

Several types of couplings are used in practice. A few are described here. Muff or sleeve coupling is shown in Figure 6.11. It is the simplest form of a permanent coupling, consisting of a steel or cast iron sleeve fitted on the ends of shaft to be connected. The sleeve is connected to the shaft by means of keys. The length of sleeve can be taken as (3.5 to 4) diameter of the shaft while the outer diameter of the muff or sleeve, *D*, is given by

$$D = d + 2\delta$$

$$D = 1.67\, d + 20 \text{ mm} \qquad \text{. . . (6.10)}$$

where *d* is the diameter of shaft in mm, δ, the thickness of the muff (Figure 6.11). However, the shear stress in the muss must be checked by treating it as a hollow shat of internal diameter *d* and external diameter *D*. The muff or sleeve coupling has the advantage of simple design and easy manufacture. However, need of perfect alignment of shafts is apparent and if not present the connection through a sleeve will induce bending stresses in the shafts. Yet another disadvantage is that while removing the sleeve must move on one of the shafts at least over a distance equal to half its length. This requires the shaft to be longer by this much amount.

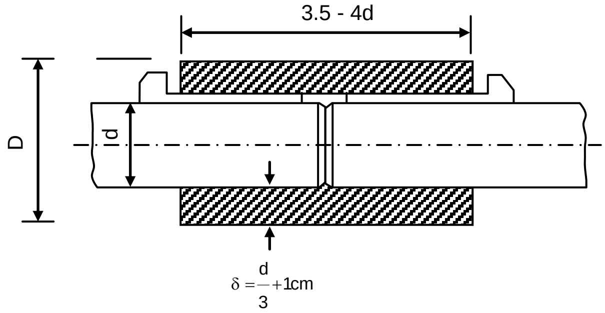
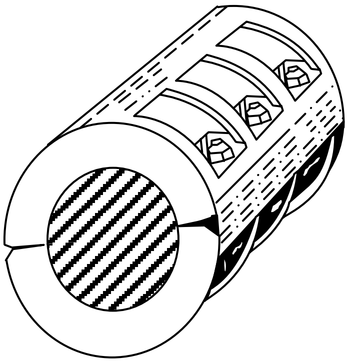
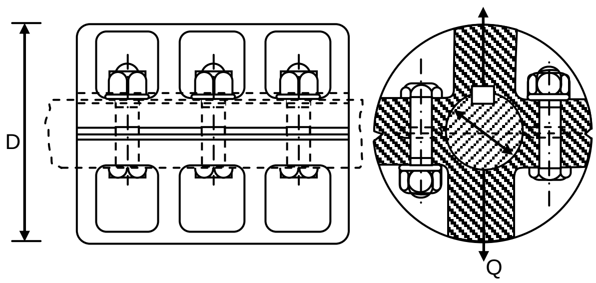


Figure 6.11 : A Sleeve Coupling

In case of split muff coupling, the sleeve is made to have two halves which are held together on two coaxial shafts by bolts. This coupling also known as clamp coupling is shown in Figure 6.12. When the bolts are tightened a compression is induced between the inner surface of sleeve and outer surface of shaft. This compressive force causes friction between the muff and the shaft which transmit the torque form one shaft to the other. In addition, a key is also used to connect the split muff with the two shafts.



(a) Split Muff with Bolts



(b) Split Muff Tightened on Two Coaxial Shafts

Figure 6.12

Split muff coupling has a distinct advantage over ordinary muff coupling as it can be removed or disassembled without disturbing the shafts.

The outer diameter of the muff, D, the length of the muff, L, and the bolt diameter d_b are the dimensions required to be determined for split muff coupling. These dimensions can be calculated from following empirical relations with shaft diameter, d.

$$D = 2.5d \quad \text{or} \quad D = 2d + 13 \text{ mm}$$

$$L = 1.5D \quad \text{or} \quad L = 3.5d \qquad \ldots (6.11)$$

$$d_b = 0.2d + 10 \text{ mm}$$

The dimensions of the key can be calculated by strength consideration or selected from standards. Such standards will be described later in this unit. Even if the bolt diameter in split muff coupling is calculated from last of Eq. (6.11) it will be worthwhile to check compression force and consequent frictional torque which results from tightening of these bolts.

6.6.1 Flange Coupling

Flange coupling, as was mentioned earlier is used to connect two strictly coaxial shafts. One such coupling is shown in Figure 6.10 and details are shown in Figure 6.13. The two flanges are usually made in cast iron. These flanges are separately keyed to driving and driven shafts.

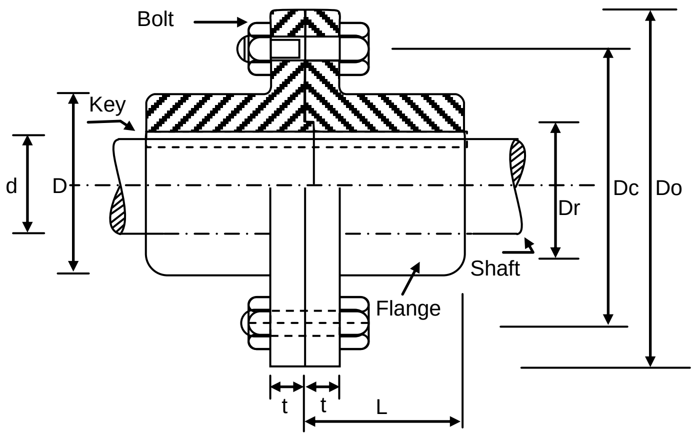


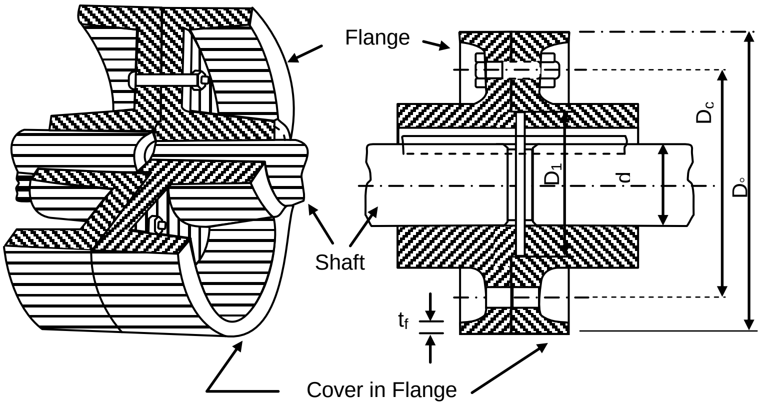
Figure 6.13 : Flange Coupling

The two flanges are identical in all respects except that one has a circular projection and other has a corresponding recess to make a register. When the two faces of flanges are brought in contact the projection fits into recess ensuring condition of coaxiality. The flanges are further connected through bolts placed near the periphery of the flanges. The faces of flanges are machine finished true right angled to the axis of shafts. The power may be transmitted by friction between the flange faces or by bolts in which case bolts will be subjected to shearing stress.

Flange couplings are often employed to transmit great torque and are largely dependable connections for shafts ranging in diameter between 18 mm to 200 mm. They are easily designed and manufactured.

Flange coupling normally refers to unprotected types as shown in Figure 6.13. The bolt head and nut, in this case are fully exposed and may present risk to operators. The bolt heads and nuts are often protected by providing cover in the flange on them as shown in Figure 6.14. This coupling is known as protected flange coupling.

While designing, the shaft diameter is calculated for transmission of torque, designated as d. The hub diameter of the flange may be calculated by treating the hub as hollow shaft but hub diameter $D = 2d$ is often adopted and is found safe. The thickness of the flange may be calculated by considering it to be in shear along the circumference where it joins the hub. However, this thickness, t, is often taken as slightly greater than diameter of the bolt.



(a) Cut View **(b) Sectional View**

Figure 6.14 : Protected Flange Coupling

The number of bolts which are placed symmetrically in a circle is determined in advance by an empirical formula

$$n = \frac{d}{50} + 3 \qquad \text{. . . (6.12)}$$

where d is the shaft diameter in mm. The number of bolts normally varies between 4 to 8.

The diameter of bolt, d_1, is determined by yet another empirical formula to obtain approximate value d_1.

$$d_1 = \frac{d}{2\sqrt{n}} \qquad \text{. . . (6.13)}$$

where d and d_1 are in mm.

The pitch circle diameter, D_c, is then determined from,

$$D_c = 2d + 2d_1 + 12 \text{ mm} \qquad \text{. . . (6.14)}$$

The diameter of bolt is then accurately determined by taking it in single shear at the interface of two flanges.

$$M_t = n \frac{\pi}{4} d_1^2 \, \tau_{s1} \frac{D_c}{2} \qquad \text{. . . (6.15)}$$

where τ_{s1} is the permissible shearing stress in bolt and M_t is the torque transmitted. The factor of safety for the bolt is higher as compared to other parts because it is subjected to sudden load at the start.

The keys in the coupling are designed in the normal manner and its depth is selected on the basis of shaft diameter which is calculated for transmission of torque only. The key dimensions for rectangular section defined $w \times h$ (width × height) can be chosen from Table 6.2.

Table 6.2 : Standard Key Section Dimensions

$w = h$ (mm^2)	d (mm)
8 × 7	28, 30
10 × 8	32, 34, 35
12 × 8	36, 37, 38, 40
14 × 9	42, 44, 45, 46, 47
16 × 10	48, 50, 52
18 × 11	55, 58, 60
20 × 12	65, 68, 70, 72
24 × 14	64, 76, 78, 80, 82, 85, 88
28 × 16	90, 92, 95, 98, 100
32 × 18	105, 110, 115
36 × 20	120, 125

Example 6.4

A driving shaft is joined with coaxial driven shaft through a muff coupling. The shaft transmits 60 kW of power at 150 rpm. Design the shaft, key and muff. Assume a factor of safety of 5 with following ultimate strength values.

Ultimate shear strength for shaft = 300 N/mm^2

Ultimate shear strength for key = 200 N/mm^2

Ultimate shear strength for muff = 50 N/mm^2

Ultimate compressive strength for key = 500 N/mm^2

Solution

If torque transmitted by the shaft is M_t Nm, power transmitted is H Watt and angular velocity is ω rad/s,

$$H = 60 \times 10^3 = M_t \, \omega = M_t \, \frac{2\pi \times 150}{60}$$

$$\therefore \quad M_t = \frac{60 \times 10^3}{5\pi} = 3819.7 \text{ Nm}$$

The stress caused by torque at outer surface of shaft of diameter, d

$$\tau_s = \frac{16 M_t}{\pi d^3} = \frac{16 \times 3819.7 \times 10^3}{\pi d^3}$$

This stresses is not to exceed permissible value $\tau_s = \frac{300}{5} = 60 \text{ N/mm}^2$.

$$\therefore \quad d = \left(\frac{19.45 \times 10^6}{60} \right)^{\frac{1}{3}} = 68.7 \text{ mm}$$

This diameter is increased by 25% to take care of weakening by key so that d = 85.9 mm say 86 mm.

From Table 6.2 choose a key with w = 24 mm and h = 14 mm. See Figure 6.15 below.

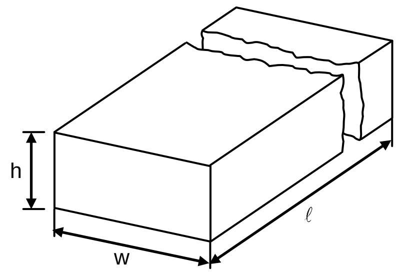


Figure 6.15 : Key

The length l of key is calculated from shear force on it.

$$\text{The shear force } = F = \frac{M_t}{\frac{d}{2}} = \frac{3819.7}{\frac{86}{2}} 10^3 = 88.83 \times 10^3 \text{ N}$$

The shear area = $w \, . \, l = 24 \, l$ mm^2

$$\text{The permissible shear stress } = \frac{200}{5} = 40 \text{ N/mm}^2$$

$$\therefore \quad \frac{88.83 \times 10^3}{24 l} = 40$$

or $$l = \frac{2.22 \times 10^3}{24} = 92.53 \text{ mm}$$

The key has to be slightly less than the half muff length. The muff length = 3.5 *d* to 4 *d*, i.e. 301 mm to 344 mm. Let's take muff length 301 mm, half of which is 150.5 mm hence, key length of 140 mm is safe.

We check height of the key against crushing under same force that causes shearing.

$$\sigma_c \times \frac{h}{2} \times l = F = 88.83 \times 10^3 \text{ N}$$

$$\therefore \quad \sigma_c = \frac{88.83 \times 10^3}{7 \times 140} = 90.6 \text{ N/mm}^2$$

Permissible compressive stress $= \frac{500}{5} = 100 \text{ N/mm}^2$

Thus, key is safe in crushing.

The muff is designed as hollow shaft with internal diameter as the diameter of the shaft. The muff will transmit same power or torque as shaft.

With D as outside diameter

$$J = \frac{\pi}{32} (D^4 - d^4)$$

$$\frac{\tau}{\frac{D}{2}} = \frac{M_t}{J} = \frac{3819.7 \times 10^3}{\frac{\pi (D^4 - d^4)}{32}}$$

The permissible shear stress in muff $= \frac{50}{5} = 10 \text{ N/mm}^2$

$$\frac{D^4 - d^4}{D} = \frac{16 \times 3819.7 \times 10^3}{10\pi} = 1945.26 \times 10^3$$

$$\therefore \quad D^4 - 86^4 = 1945.36 \times 10^3 \ D$$

$$D^4 - 1.95 \times 10^6 \ D = 54.7 \times 10^6 \qquad \text{. . . (i)}$$

This equation can be solved by trial and error and to get an idea of starting point take D = 2.5 d = 215 mm. With this value, the term on right hand side can be neglected resulting in $D = (1950 \times 10^3)^{\frac{1}{3}} = 125 \text{ mm}$.

Choose value of 130, 140, 150, 160 mm for D. Then for D = 140 mm, $D_4 = 38.4 \times 10^7$

$\therefore$ Left hand side of (i) $384 \times 10^6 - 273 \times 10^6 = 31.5 \times 10^6$

For D = 130 mm, $D^4 = 285 \times 10^6$

$\therefore$ Left hand side of (i) $285 \times 10^6 - 253.5 \times 10^6 = 31.5 \times 10^6$

For D = 135 mm, LHS $= 332 \times 10^6 - 263 \times 10^6 = 69 \times 10^6$

For D = 132 mm, LHS $= 303.6 \times 10^6 - 257.4 \times 10^6 = 46.2 \times 10^6$

For D = 133 mm, LHS $= 313 \times 10^6 - 259.4 \times 10^6 = 53.6 \times 10^6$

D = 133 mm comes closest to solution of (i).

The above trial and error method has been given to make reader familiar with such method. We would rather select the outer diameter from empirical formula.

$$D = 2d + 13 \text{ mm} = 2 \times 86 + 13 = 185 \text{ mm}$$

Example 6.5

A shaft transmitting 150 kW is to be connected to a coaxial shaft through cast iron flange coupling. The shaft runs at 120 rpm. The key and shaft are to be made of same material for which permissible shearing stress is 60 N/mm^2 and compressive strength is 120 N/mm^2. The steel bolts may be subjected to maximum shearing stress of 26 N/mm^2. Design protected type flange coupling.

Solution

Shaft Diameter d

$$H = M_t\ \omega \quad \text{or} \quad 150 \times 10^3 = M_t \times \frac{2\pi \times 120}{60}$$

$$\therefore \qquad M_t = \frac{150 \times 10^3}{12.57} = 12 \times 10^3 \text{ Nm} = 12 \times 10^6 \text{ Nmm}$$

For the shaft

$$\tau = \frac{16 M_t}{\pi d^3} = 60 \text{ N/mm}^2$$

$$\text{or} \qquad d = \left(\frac{16 \times 12 \times 10^6}{\pi \times 60} \right)^{\frac{1}{3}} = 100.6 \text{ mm}$$

Increase diameter by 25% to take care of keyway.

$$\therefore \qquad d = 125 \text{ mm} \qquad \text{. . . (i)}$$

Bolt Diameter d_1

Let there be n bolts clamping two flanges and let each bolt be subjected to shearing stress τ_1. The force produced tangential to pitch circle of bolts (The diameter of pitch circle is D_C from Figure 6.13)

$$F = n \frac{\pi}{4} d_1^2\ \tau_1$$

The torque produced by F must be equal to torque transmitted by the shaft.

$$\therefore \qquad M_t = F \frac{D_c}{2} = n \frac{\pi}{4} d_1^2\ \tau_1 \frac{D_c}{2} \qquad \text{. . . (ii)}$$

From Eq. (6.13)

$$n = \frac{d}{50} + 3 = \frac{125}{50} + 3 = 5.5 \text{ say } 6$$

Also from Eq. (8.24)

$$d_1 = \frac{d}{2\sqrt{n}} = \frac{125}{2\sqrt{6}} = 25.5 \text{ mm}$$

D_c can be obtained form Eq. (6.14)

$$D_c = 2d + 2d_1 + 12 \text{ mm}$$

$$= 2 \times 125 + 2 \times 25.5 + 12 = 313 \text{ mm}$$

We calculate RH side of (ii) by using values of d_1, D_c and $\tau_1 = 26$ N/mm^2.

$$M_t = 6 \times \frac{\pi}{4} (25.5)^2 \times 26 \times \frac{313}{2} = 12.47 \times 10^6 \text{ Nmm}$$

Since this value is greater than torque transmitted, 12×10^6 Nmm,

$$n = 6,\ d_1 = 25.5 \text{ mm},\ D_c = 313 \text{ mm} \quad \ldots \text{(iii)}$$

are acceptable values.

Hub Diameter D

The hub diameter can be taken as $2d$, with internal diameter = d. Then treating hub as hollow shaft under torque M_t, the shear stress should be less than 6.6 N/mm^2 (shear stress in C.I).

$$M_t = \left[\frac{\pi}{32} \frac{(D^4 - d^4)}{\frac{D}{2}} \right] \tau_2 \quad \text{with} \quad D = 2d = 250 \text{ mm}$$

$$\therefore \quad \tau_2 = \frac{16 \times 12 \times 10^6 \times 250}{\pi (250^4 - 125^4)} = \frac{15.3 \times 10^9}{36.6 \times 10^8} = 4.18 \text{ N/mm}^2$$

This stress is less then 6.6 N/mm^2, hence, D = 250 mm is safe.

$$\therefore \quad D = 250 \text{ mm} \quad \ldots \text{(iv)}$$

Length of Hub, L

Length of the hub is equal to length of the key.

From Table 6.2 for shaft diameter of 125 mm, find w = 36 mm, h = 20 mm.

You may also choose a square key with $w = h = \frac{d}{4} = 31.25$ mm .

Shear stress in key is same as in shaft, τ = 60 N/mm^2.

$$\therefore \quad M_t = l \times w \times \tau \frac{d}{2} = l \times 36 \times 60 \times 62.5,\ M_t = 12 \times 10^6 \text{ Nmm}$$

$$\therefore \quad l = \frac{12 \times 10^6}{36 \times 60 \times 62.5} = 89 \text{ mm} \quad \ldots \text{(v)}$$

Thickness of Flange

There is possibility of failure by shear along the circumference where flange joint the hub. If t is the thickness of the flange, the area over which shear may occur is $\pi D t$. The shear force will be $\pi D t \tau_3$, τ_3 being the permissible shearing stress in cast iron flange. This, will cause the torque equal to the torque transmitted by the shaft

$$\therefore \quad \pi D t \tau_3 \frac{D}{2} = M_t$$

$$\therefore \quad t = \frac{2 \times 12 \times 10^6}{\pi (250)^2 \times 6.6} = 18.5 \text{ mm}$$

The bolts in holes of flange may be crushed. Of course the hole surface may also be crushed but if bolts are safe then the hole surface will be safe since the CI is stronger than steel in compression.

The area resisting crushing is $d_1\, t$ and force in n bolts is $n\, d_1\, t\, \sigma_c$ at a radius of $\frac{D_c}{2}$. Thus, the torque is

$$n\, d_1\, t\, \tau_3\, \frac{D_c}{2} = 6 \times 25.5 \times 18.5 \times 120 \times \frac{313}{2} = 53 \times 10^6 \text{ Nmm}$$

This torque is much larger than 12×10^6 Nmm, and hence dimensions are safe.

Thus, $t = 18.5$ mm . . . (vi)

Other Dimensions

The outer diameter of flange is calculated from

$$D_o = 2D_c - D = 2 \times 313 - 250 = 376 \text{ mm} \quad \text{. . . (vii)}$$

The diameter of register, $D_r = \frac{D_o}{2} = 188$ mm . . . (viii)

Thickness of the protective cover on the top of the flange

$$t_f = \frac{d}{4} \text{ or } t\text{, Choose } t_f = t = 18.5 \text{ mm} \quad \text{. . . (ix)}$$

The extension of protection should be 5 mm greater than nut height on both flanges.

Summary of Results

Shaft diameter	d = 125 mm	. . . (i)
Bolt diameter	d_1 = 25.5 mm	. . . (ii)
Number of bolts	n = 6	. . . (iii)
Pitch circle diameter of bolts	D_c = 313 mm	. . . (iv)
Hub diameter	D = 250 mm	. . . (v)
Length of hub	L = 89 mm	. . . (vi)
Key dimensions	w = 36 mm, h = 20 mm, L = 89 mm	. . . (vii)
Thickness flange	t = 18.5 mm	. . . (viii)
Outer diameter of flange	D_o = 376 mm	. . . (ix)
Diameter of register	D_r = 188 mm	. . . (x)
Thickness of protective cover	t_f = 18.5 mm	. . . (xi)

SAQ 1

(a) Sketch a muff coupling and identify its advantages and disadvantages.

(b) Sketch a flange coupling and mention how strength of bolts and thickness of the flange can be calculated.

(c) Mention materials for shaft, flange, keys and bolt.

(d) Show register in flange. What purpose does it serve?

(e) Design and draw a flange coupling, to connect two coaxial shafts of an electric motor and worm and worm wheel reducer. The shafts transmit 7 kW of power at 300 rpm. The permissible stresses are :

Shearing stress in shaft = 50 N/mm^2

Shearing stress in key = 25 N/mm^2

Shearing stress in coupling = 3 N/mm^2

Shearing stress in bolt = 25 N/mm^2

The results must consist of shaft diameter (d), which has to be increased by 25% to take care of keyway, number of bolts (n), diameter of bolts (d_1), pitch circle diameter of bolts (D_c), diameter of hub (D), length of hub (L), assume square key of size $\frac{d}{4}$, thickness of flange (t), outside flange diameter (D_o).

6.7 SUMMARY

Shaft is an important machine element and transmits power. The keyways becomes essential feature of shafts because some part like gear or pulley has to be attached on it to transmit power. The keys are standardised and can be selected from relevant table. There is yet simpler method to use a square key of depth $\frac{1}{4}$ of diameter of shaft.

Couplings connect coaxial shafts. They are formed by two discs attached to shafts through key and jointed by bolts, parallel to shaft axis. The discs are made as flanges integral with the hub. The flanges are often made in cast iron. Muff couplings are thick cylinders which could be used as sleeves or split to be bolted around the shafts. The driving force in muff coupling is friction between the inner surface of muff and outer surface of shaft. The muff can be a single piece sleeve keyed to shafts or split in halves which are tightened by the bolts. The muff is made in cast iron.

6.8 KEY WORDS

Shaft	:	A cylindrical machine part which transmits power and is subjected to BM and torque.
Key	:	A part of rectangular cross-section which connects gear or pulley to shaft.
Coupling	:	Device to connect coaxial shafts.
Muff	:	A hollow cylinder which may or may not be split along central line.
Flange Coupling	:	Flanges integral with hub which connects to shaft via key. Plane surfaces of two flanges on two axial shafts contact. The flanges are connected through bolts.

6.9 ANSWERS TO SAQs

SAQ 1

(e) Follow Example 6.4, Section 6.6 and match dimension with those shown in Figure 6.16.

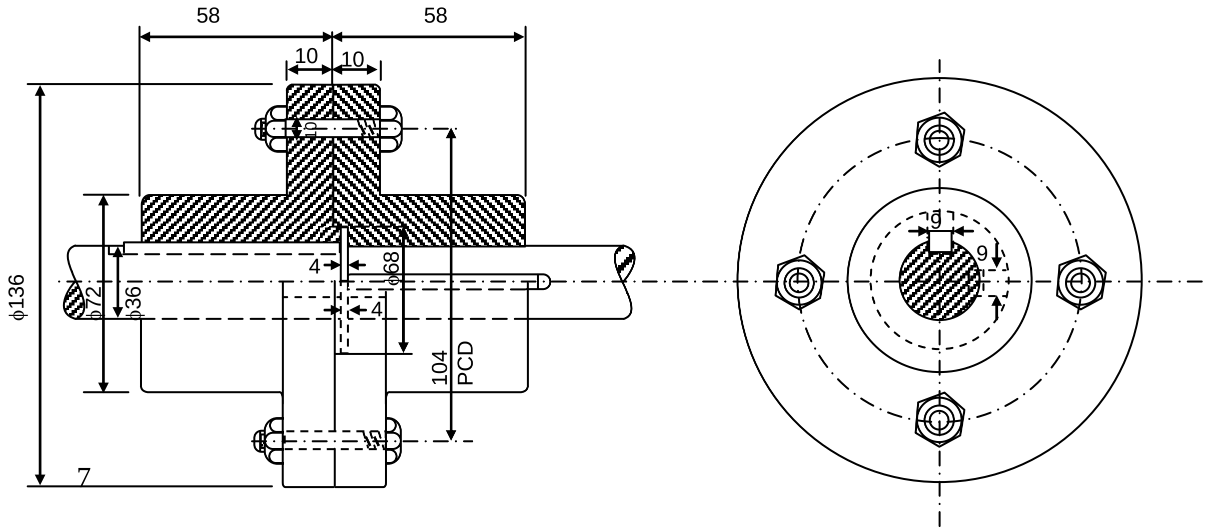


Figure 6.16